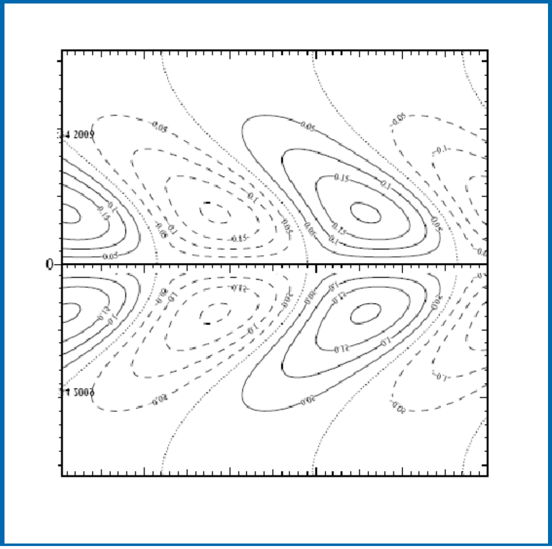
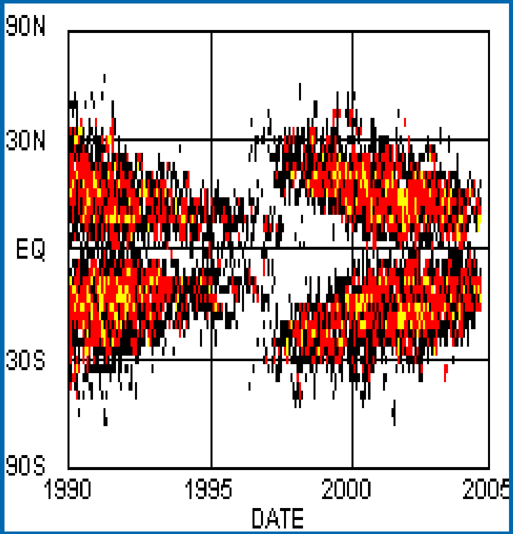
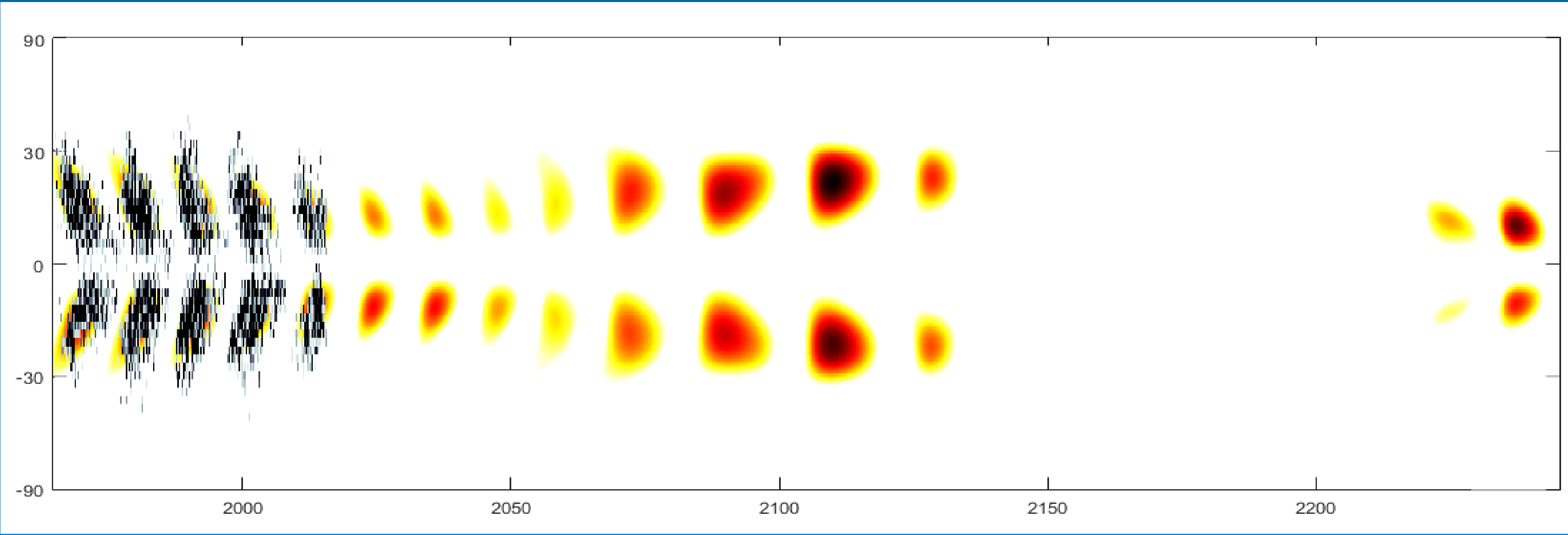
Крым 2019

INTRODUCTION TO DYNAMO THEORY



N. KLEEORIN





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Induction equation – from Maxwell equation

$$\nabla \cdot \mathbf{D} = 4\pi\rho_e$$

$$\nabla \cdot \mathbf{B} = 0$$

$$\nabla \times \mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla \times \mathbf{H} = \frac{4\pi}{c}\mathbf{j} + \frac{1}{c}\frac{\partial \mathbf{D}}{\partial t}$$

$$\frac{\partial \rho_e}{\partial t} + \nabla \cdot \mathbf{j} = 0$$

Induction equation – from Maxwell equation

$$\nabla \cdot \mathbf{D} = 4\pi\rho_e \qquad \mathbf{D} = \varepsilon \mathbf{E};$$

$$\nabla \cdot \mathbf{B} = 0 \qquad \mathbf{B} = \mu \mathbf{H}$$

$$\nabla \times \mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t} \qquad \mathbf{E}' = \mathbf{E} + \frac{\mathbf{v}}{c} \times \mathbf{B}$$

$$\nabla \times \mathbf{H} = \frac{4\pi}{c}\mathbf{j} + \frac{1}{c}\frac{\partial \mathbf{D}}{\partial t} \qquad \mathbf{j} = \sigma \mathbf{E}' = \sigma\left(\mathbf{E} + \frac{\mathbf{v}}{c} \times \mathbf{B}\right)$$

$$\frac{\partial \rho_e}{\partial t} + \nabla \cdot \mathbf{j} = 0$$

Induction equation – from Maxwell equation

- **Induction equation**

$$\nabla \cdot \mathbf{D} = \cancel{4\pi\rho_e} = 0 \qquad \mathbf{D} = \mathbf{E};$$

$$\nabla \cdot \mathbf{B} = 0 \qquad \mathbf{B} = \mathbf{H}$$

$$\nabla \times \mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t} \qquad \mathbf{E}' = \mathbf{E} + \frac{\mathbf{v}}{c} \times \mathbf{B}$$

$$\nabla \times \mathbf{H} = \frac{4\pi}{c}\mathbf{j} + \cancel{\frac{1}{c}\frac{\partial \mathbf{D}}{\partial t}} \qquad \Leftarrow L_{sist} \ll cT_{sist}$$

$$\frac{\partial \rho_e}{\partial t} + \nabla \cdot \mathbf{j} = 0$$

Induction equation – from Maxwell equation

$$\nabla\times\mathbf{E}=-\frac{1}{c}\frac{\partial\mathbf{B}}{\partial t}\qquad \mathbf{E}'=\mathbf{E}+\frac{\mathbf{v}}{c}\times\mathbf{B}$$

$$\nabla\times\mathbf{B}\approx\frac{4\pi}{c}\mathbf{j}\qquad \mathbf{j}=\sigma\mathbf{E}'=\sigma\left(\mathbf{E}+\frac{\mathbf{v}}{c}\times\mathbf{B}\right)$$

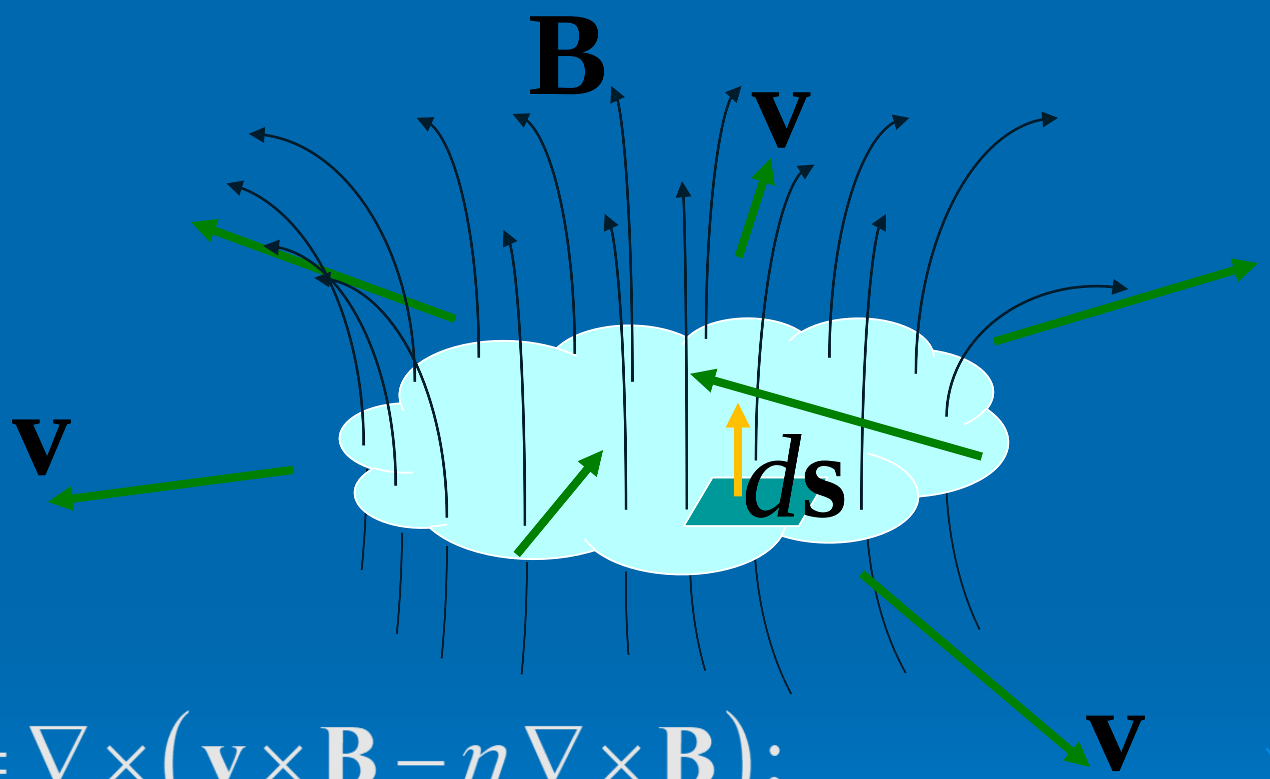
$$\mathbf{E}'=\frac{\mathbf{j}}{\sigma}\approx\frac{c\nabla\times\mathbf{B}}{4\pi\sigma}\Rightarrow\mathbf{E}=\mathbf{E}'-\frac{\mathbf{v}}{c}\times\mathbf{B}\Rightarrow\mathbf{E}\approx\frac{c\nabla\times\mathbf{B}}{4\pi\sigma}-\frac{\mathbf{v}}{c}\times\mathbf{B}$$

$$\nabla\times\left(\frac{c\nabla\times\mathbf{B}}{4\pi\sigma}-\frac{\mathbf{v}}{c}\times\mathbf{B}\right)=-\frac{1}{c}\frac{\partial\mathbf{B}}{\partial t}$$

$$\frac{\partial\mathbf{B}}{\partial t}=\nabla\times\left(\mathbf{v}\times\mathbf{B}-\eta\nabla\times\mathbf{B}\right);\qquad \eta=\frac{c^2}{4\pi\sigma}$$

$$\frac{\partial\mathbf{B}}{\partial t}=\nabla\times\left(\mathbf{v}\times\mathbf{B}\right)+\eta\Delta\mathbf{B}$$

Induction equation – Properties



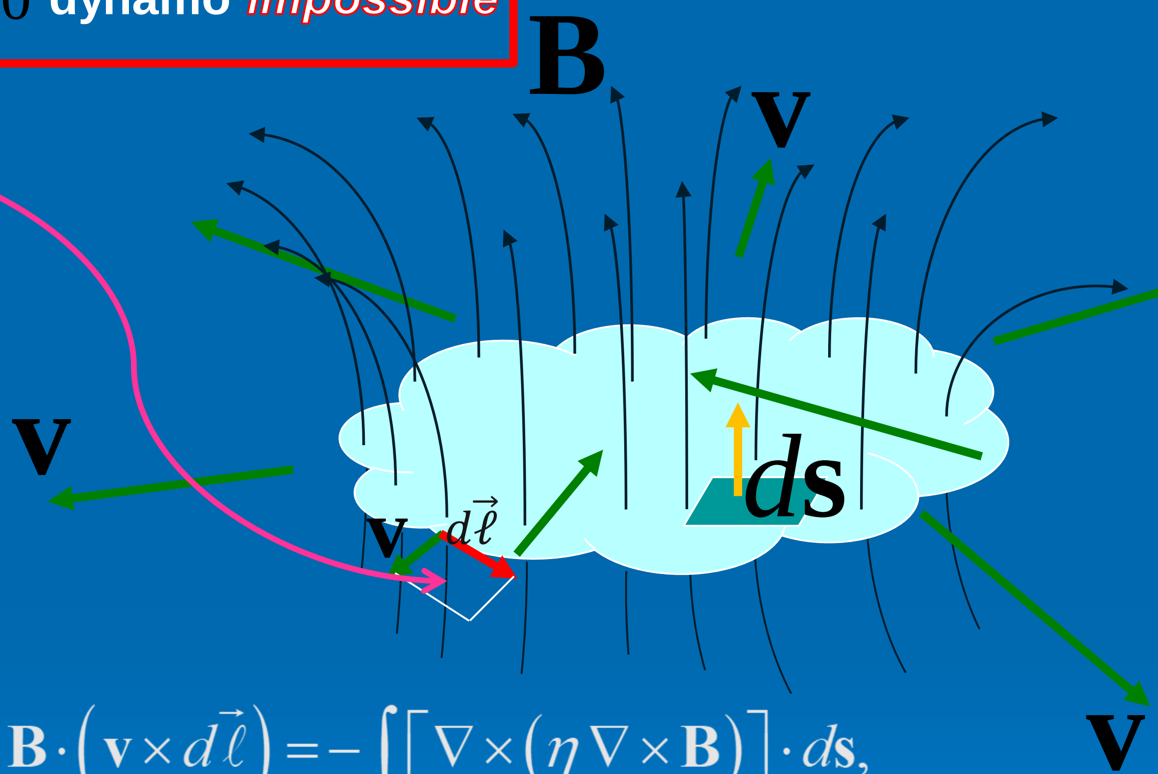
$$\int \cdot d\mathbf{s} \left| \frac{\partial \mathbf{B}}{\partial t} = \nabla \times \left(\mathbf{v} \times \mathbf{B} - \eta \nabla \times \mathbf{B} \right); \right.$$

$$\int \frac{\partial \mathbf{B}}{\partial t} \cdot d\mathbf{s} = \oint \left(\mathbf{v} \times \mathbf{B} \right) \cdot d\vec{\ell} - \int \nabla \times \left(\eta \nabla \times \mathbf{B} \right) \cdot d\mathbf{s};$$

Induction equation – Properties

If $\eta = 0$ **dynamo** ***impossible***

$$\frac{\delta \mathbf{s}}{dt} = \mathbf{v} \times d\vec{\ell}$$



$$\int \frac{\partial \mathbf{B}}{\partial t} \cdot d\mathbf{s} + \oint \mathbf{B} \cdot \left(\mathbf{v} \times d\vec{\ell} \right) = -\int \left[\nabla \times \left(\eta \nabla \times \mathbf{B} \right) \right] \cdot d\mathbf{s},$$

$$\Downarrow$$

$$\int \frac{\partial \mathbf{B}}{\partial t} \cdot d\mathbf{s} + \oint \mathbf{B} \cdot \frac{\delta \mathbf{s}}{dt} \equiv \frac{d}{dt} \int \mathbf{B} \cdot d\mathbf{s} = \frac{d\Phi_{\mathbf{B}}}{dt} = -\int \left[\nabla \times \left(\eta \nabla \times \mathbf{B} \right) \right] \cdot d\mathbf{s},$$

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times \left(\mathbf{v} \times \mathbf{B} - \eta \nabla \times \mathbf{B} \right);$$

Alpha-Omega Dynamo (Mean-Field Approach)

- **Induction equation for mean magnetic field:**

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times \left(\mathbf{U} \times \mathbf{B} + \boldsymbol{\varepsilon} - \eta \nabla \times \mathbf{B} \right)$$

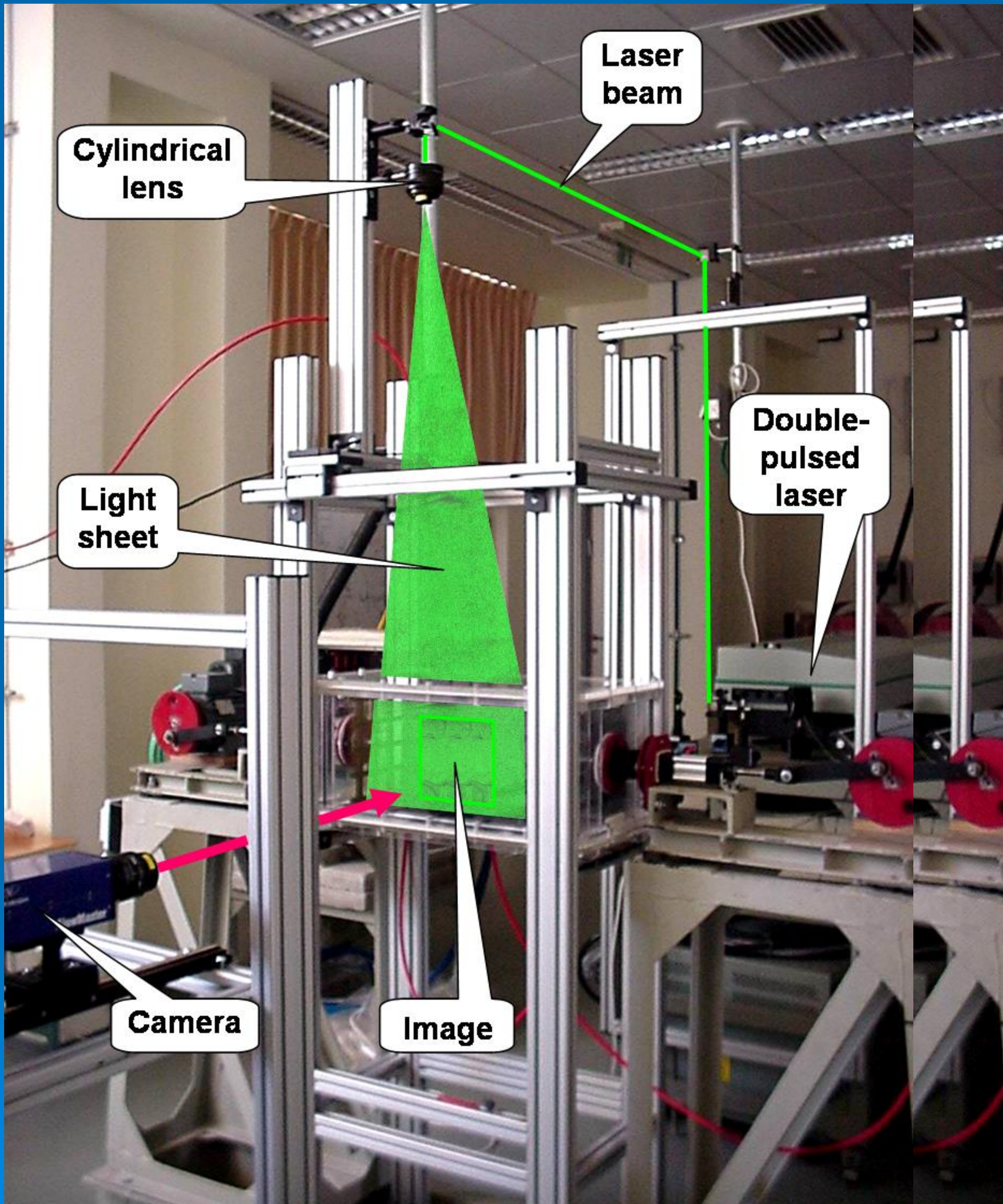
- **Electromotive force:** Shteenbeck, Krause and Raedler (1966)

$$\mathbf{v} = \mathbf{U} + \mathbf{u}; \ \mathbf{B} \rightarrow \mathbf{B} + \mathbf{b}$$

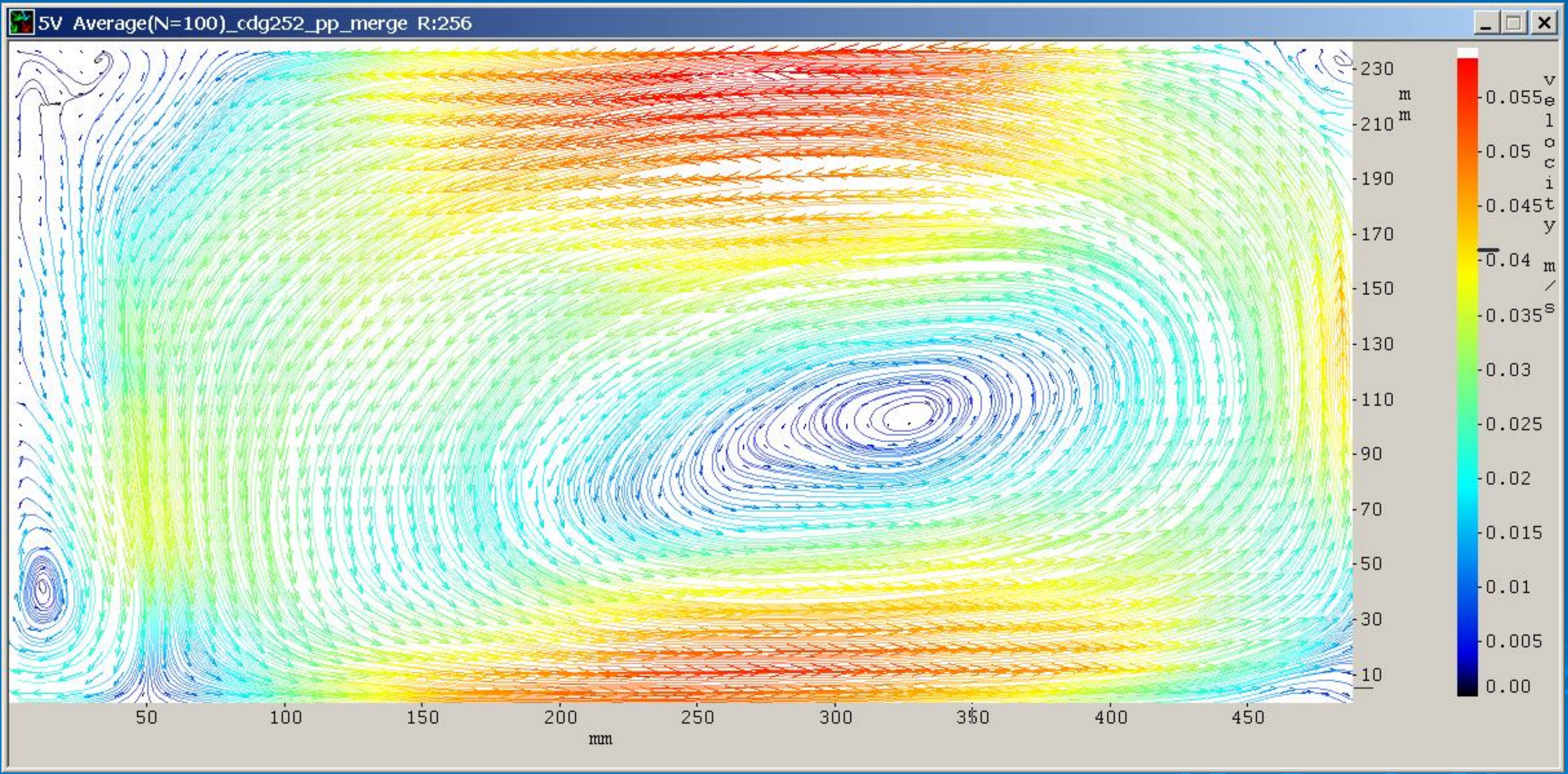
$$\boldsymbol{\varepsilon} \equiv \langle \mathbf{u} \times \mathbf{b} \rangle = \alpha \mathbf{B} - \eta_T \nabla \times \mathbf{B} + \ldots$$

$$\alpha = -\frac{\tau}{3} \langle \mathbf{u} \cdot \mathbf{rot}\ \mathbf{u} \rangle; \quad \eta_T = \frac{\tau \langle \mathbf{u}^2 \rangle}{3}$$

Experimental set – up



Laboratory Turbulent Convection



Before averaging

After averaging

Mean-Field Approach

- **Induction equation for mean magnetic field:**

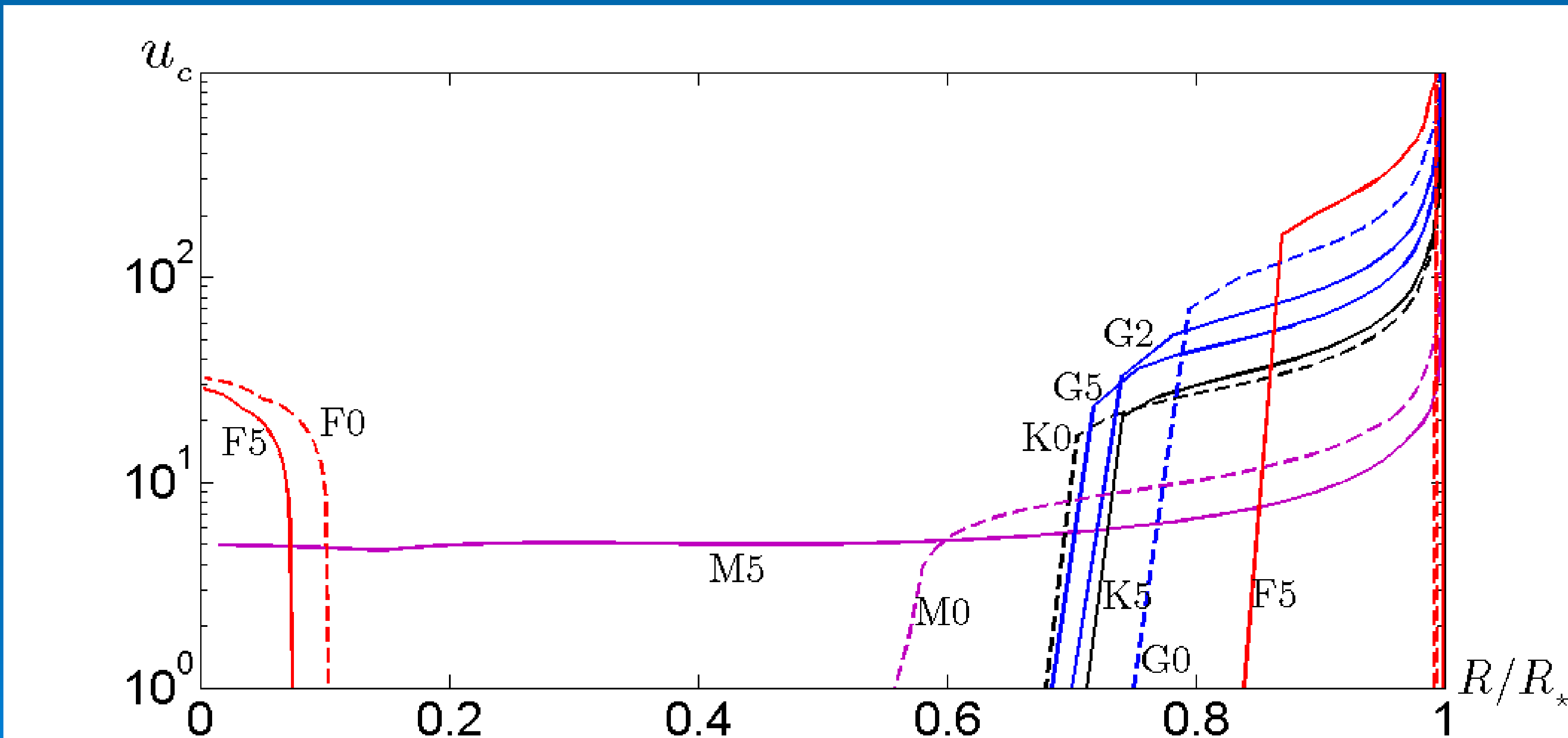
$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times \left(\mathbf{U} \times \mathbf{B} + \boldsymbol{\varepsilon} - \eta \, \nabla \times \mathbf{B} \right)$$

- **Electromotive force:**

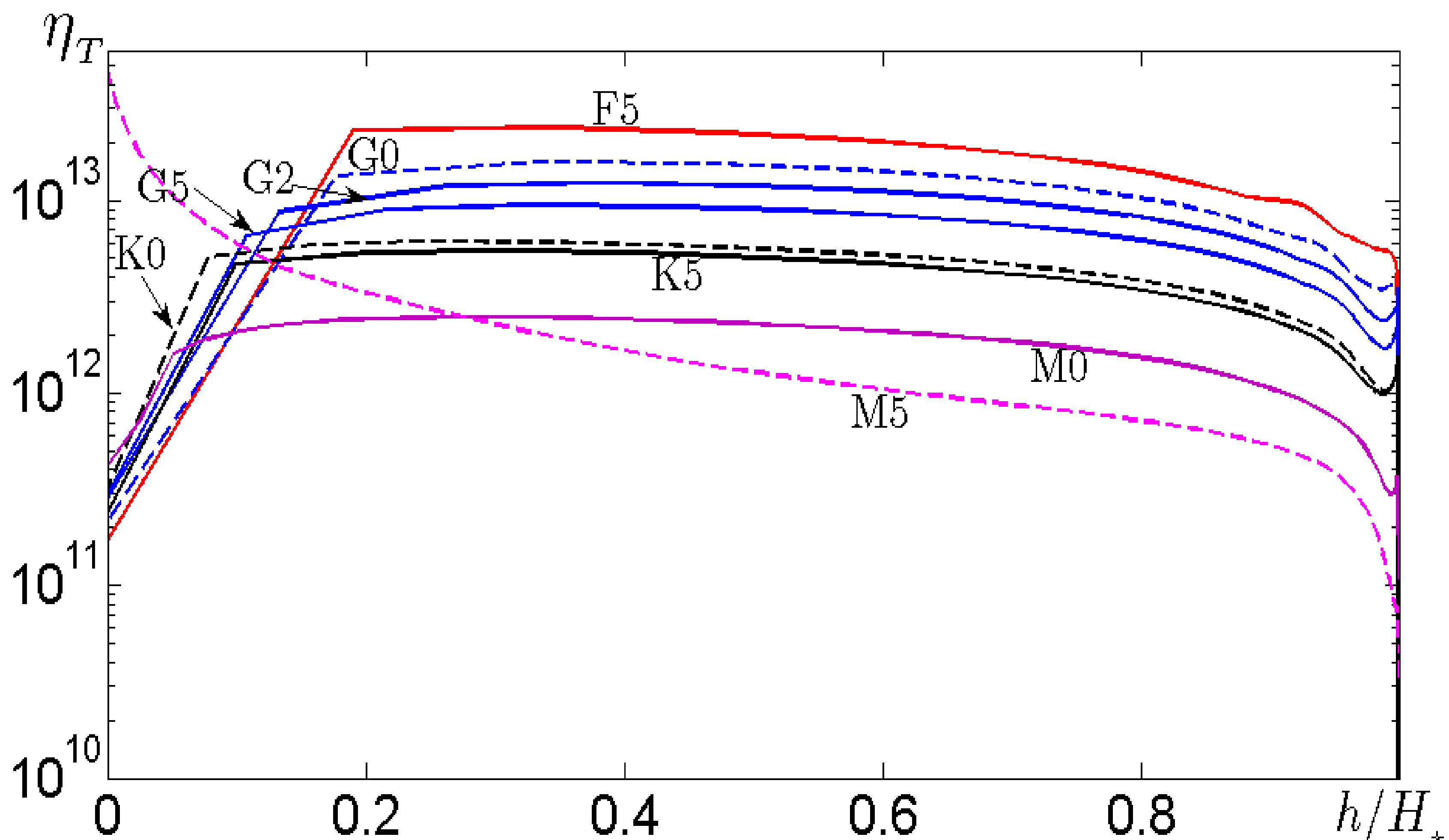
$$\boldsymbol{\varepsilon} \equiv \left\langle \mathbf{u} \times \mathbf{b} \right\rangle = \alpha \, \mathbf{B} - \eta_T \nabla \times \mathbf{B} + \ldots$$

$$\alpha = -\frac{\tau}{3} \left\langle \mathbf{u} \cdot \mathbf{rot}\ \mathbf{u} \right\rangle; \;\; \eta_T = \frac{\tau}{3} \ell_0 u_c$$

Convective velocity in a star of main sequence

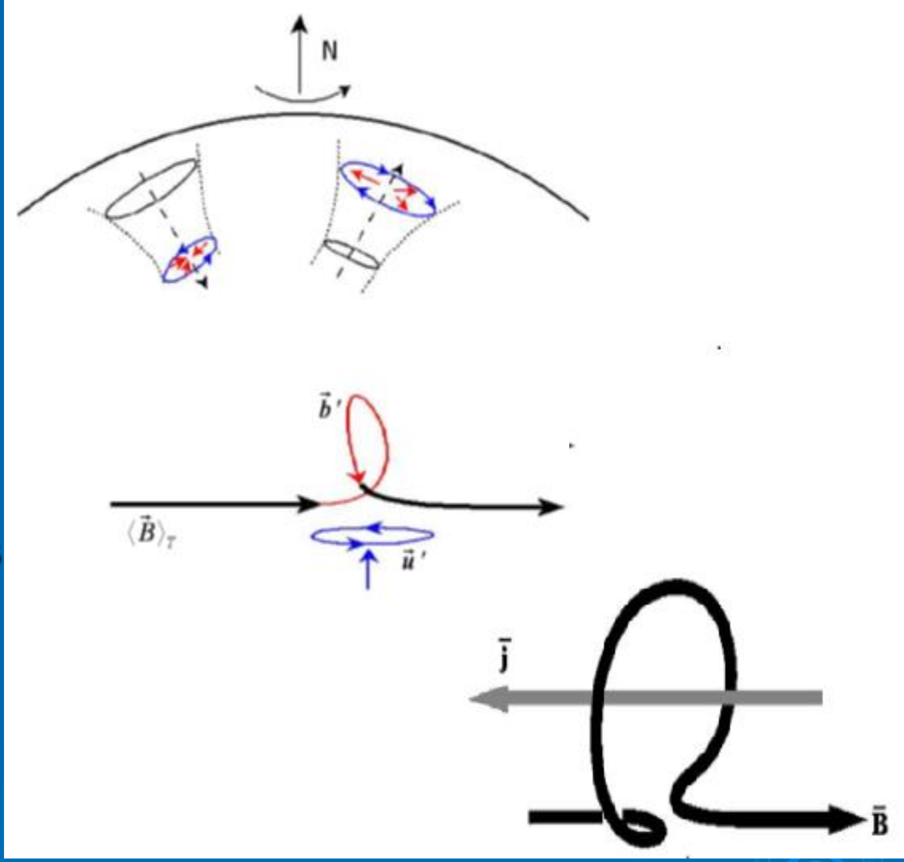


Turbulent diffusivity in a star of main sequence



Physics of the alpha-effect

- The α -effect is related to the hydrodynamic helicity in an inhomogeneous turbulence.
- The deformations of the magnetic field lines are caused by upward and downward rotating turbulent eddies.
- The inhomogeneity of turbulence breaks a symmetry between the upward and downward eddies.
- Therefore, the total effect of the upward and downward eddies on the mean magnetic field does not vanish and it creates the mean electric current parallel to the original mean magnetic field.



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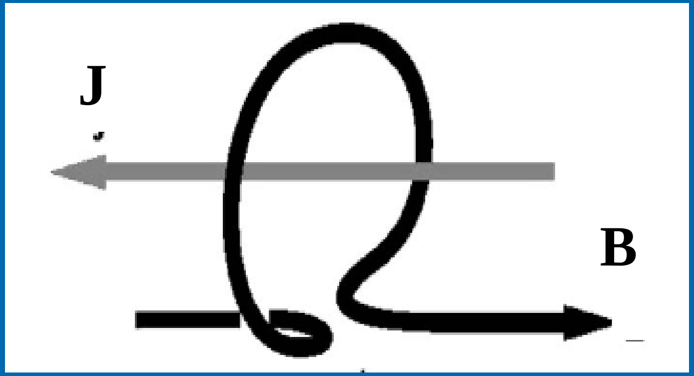
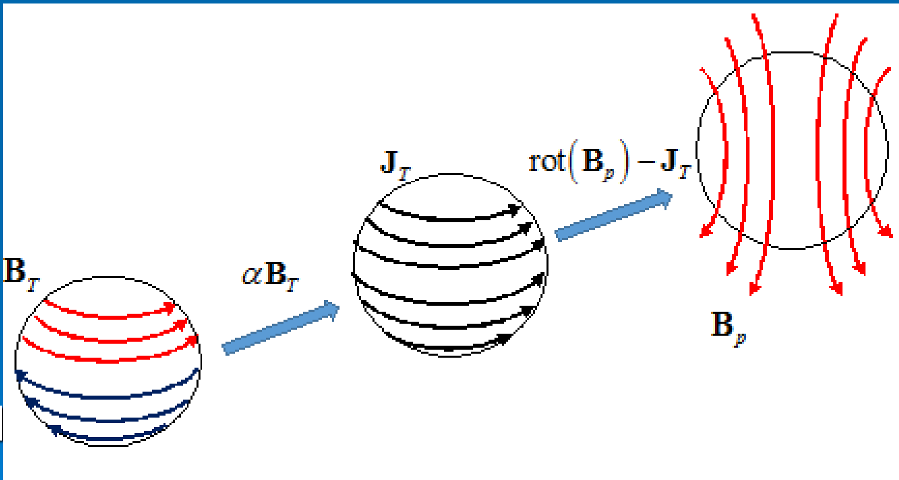




Alpha-effect in Dynamo (Mean-Field Approach)

- **Induction equation for mean magnetic field:**

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times \left(\mathbf{U} \times \mathbf{B} + \alpha\, \mathbf{B} - (\ \eta_T + \eta\ ) \nabla \times \mathbf{B} \right)$$

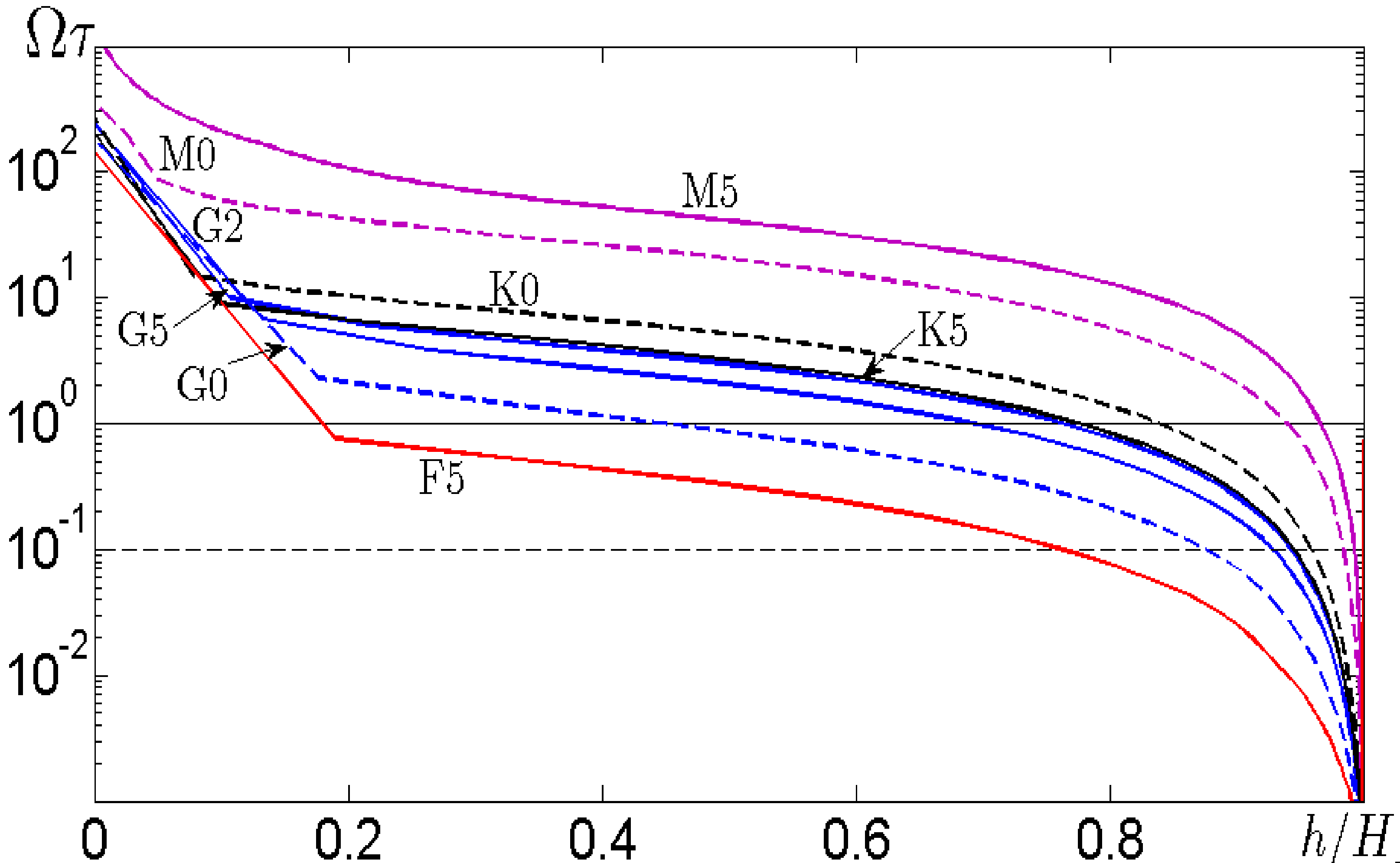
- **Electromotive force:**

$$\boldsymbol{\varepsilon} \equiv \langle \mathbf{u} \times \mathbf{b} \rangle = \alpha\, \mathbf{B} - \eta_T \nabla \times \mathbf{B} + \ldots \quad \nabla \times \mathbf{B} \approx \frac{4\pi}{c} \mathbf{J}$$

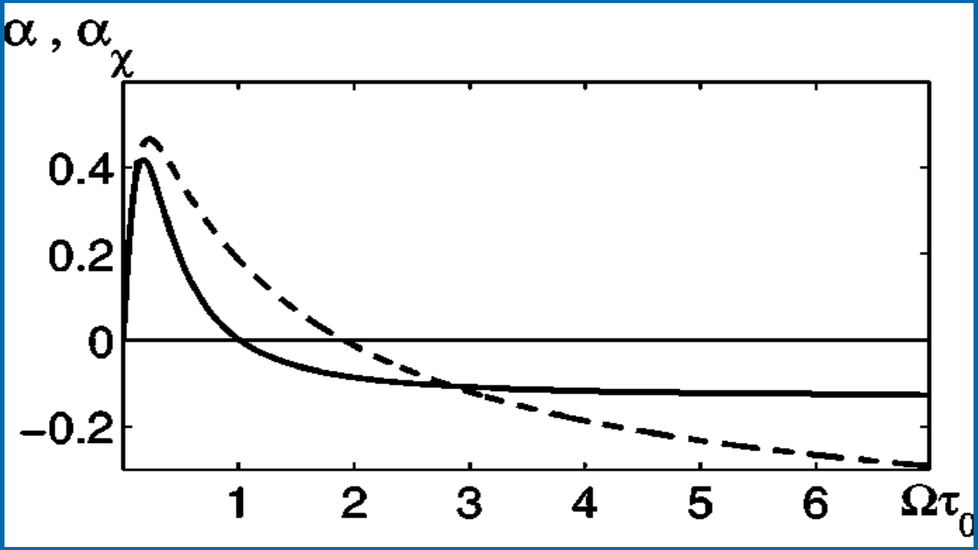
$$\mathbf{J} = \sigma_{eff} \left[\mathbf{E} + \frac{1}{c} \left(\mathbf{U} \times \mathbf{B} + \alpha\, \mathbf{B} \ \right) \right]; \quad \sigma_{eff} = \frac{c^2}{4\pi (\ \eta_T + \eta\ )}$$

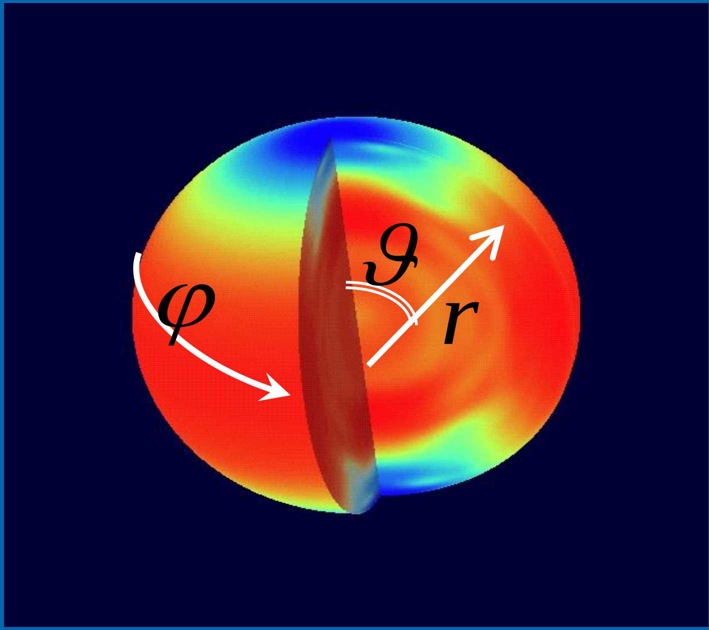
$$\eta_T = \frac{\ell_0 u_c}{3}; \quad \alpha = -\frac{\tau}{3} \langle \mathbf{u} \cdot \mathbf{rot}\ \mathbf{u} \rangle \approx \frac{\Omega \ell_0^2}{3 L_\rho} \cos(\vartheta); L_\rho^{-1} = -\frac{1}{\rho} \frac{\partial \rho}{\partial r}, \Omega\tau \ll 1$$

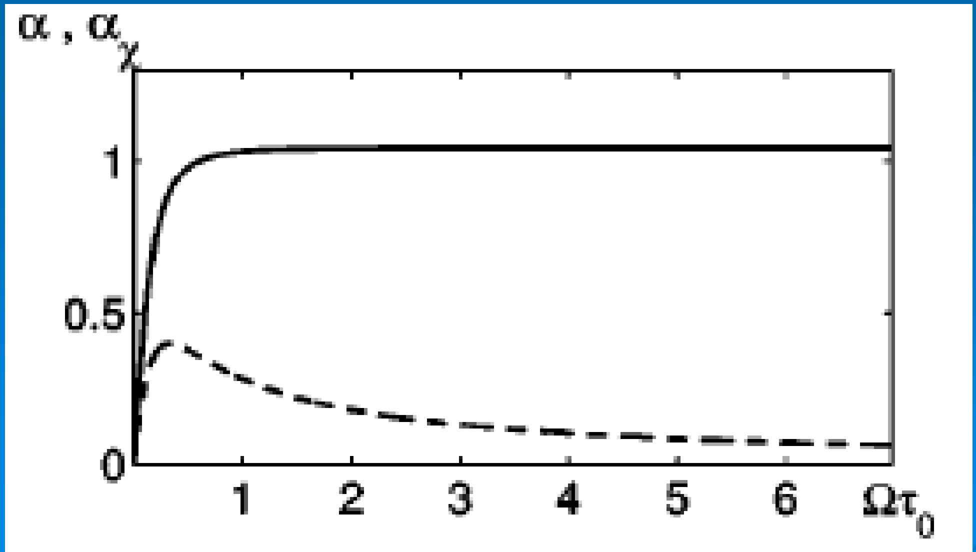
Rossby-parameters $\Omega_* \tau_*$ for main sequence stars



Radial profiles of the **alpha-effect**





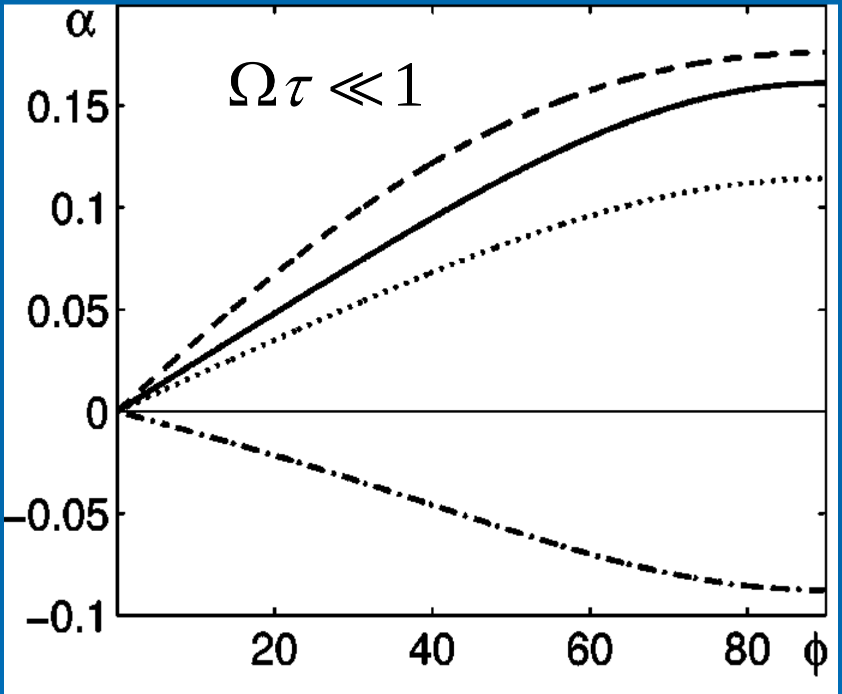


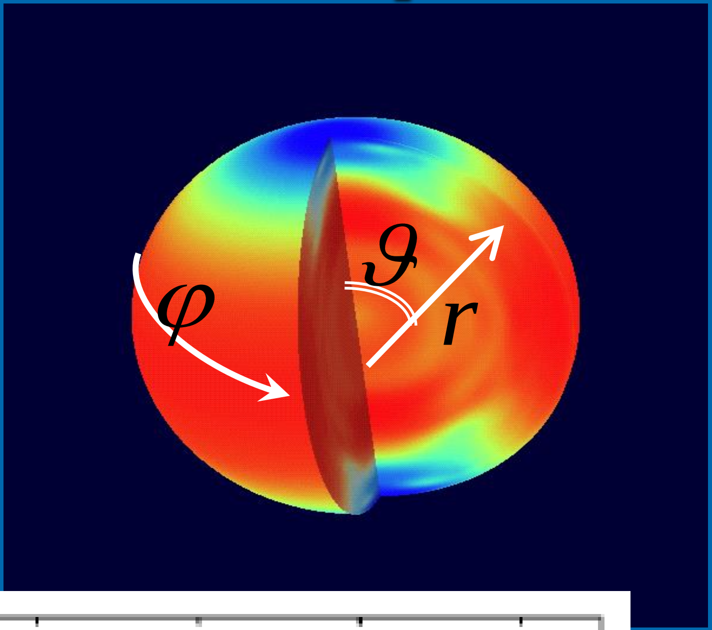
Kichatinov, L. L. & Rudiger, G. (1993)

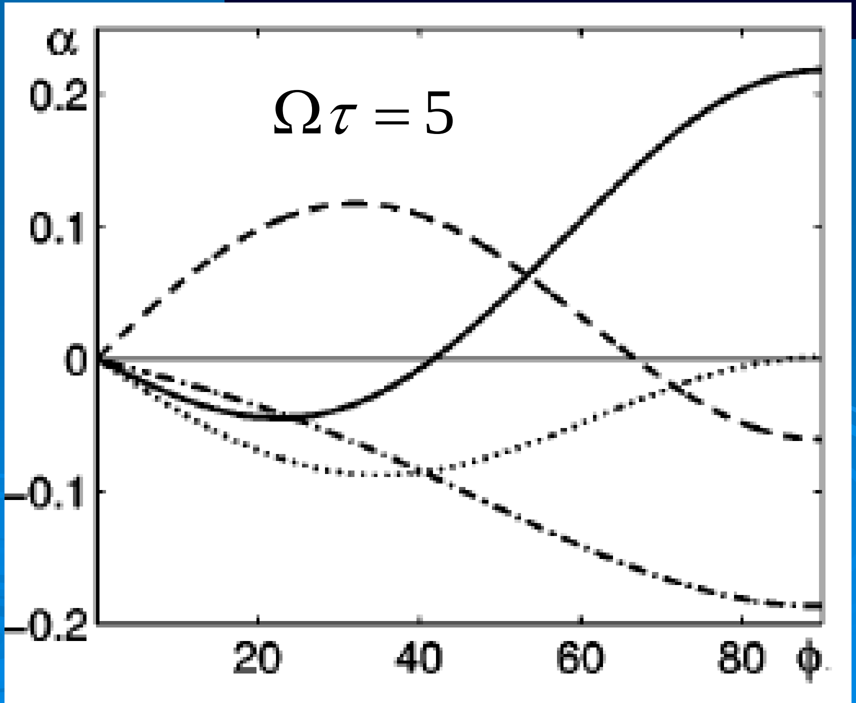
Kleeorin & Rogachevskii (2003)

Latitudinal profiles of the **alpha-effect**

$$\phi = \frac{\pi}{2} - \vartheta$$



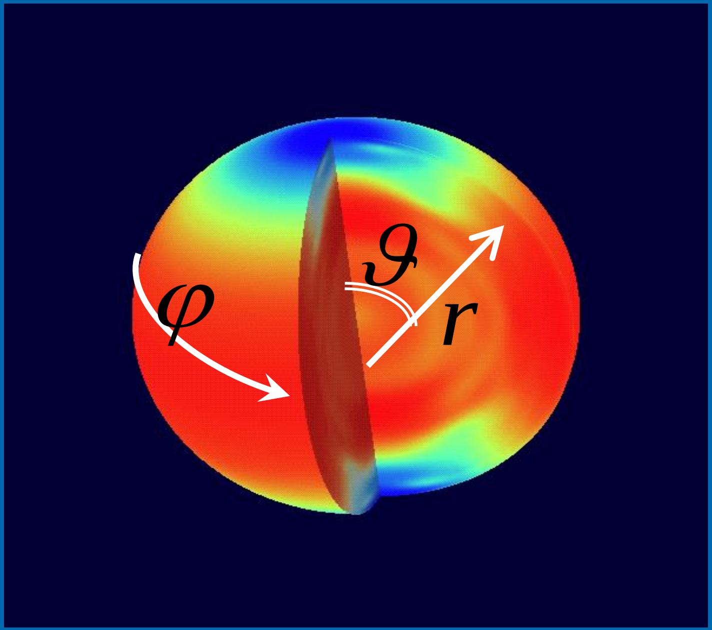


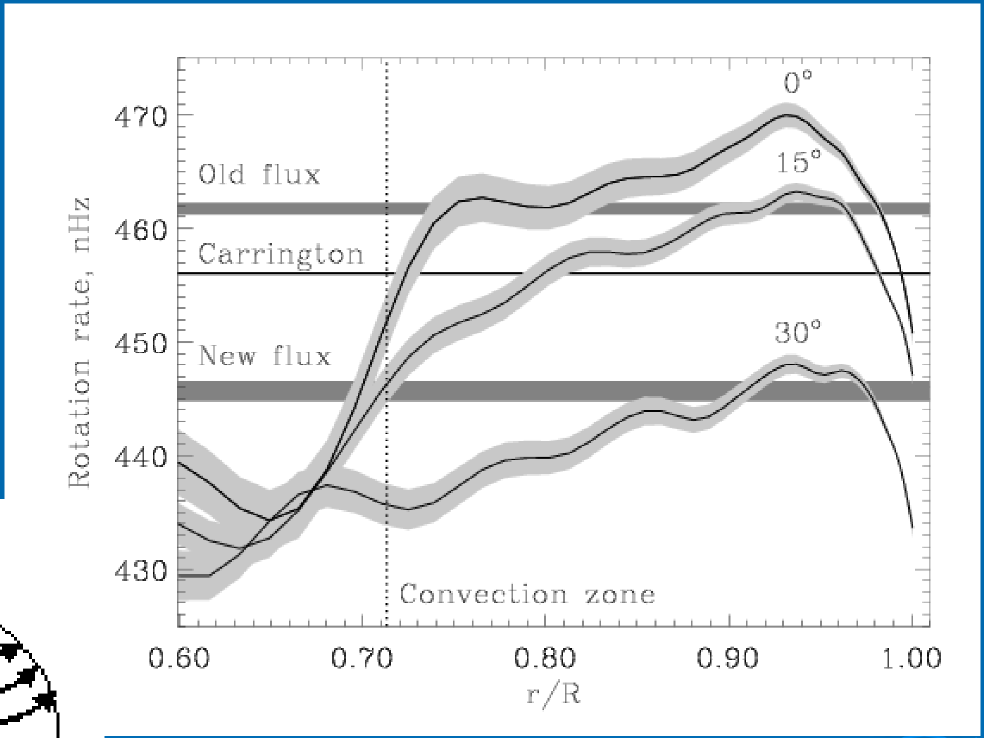


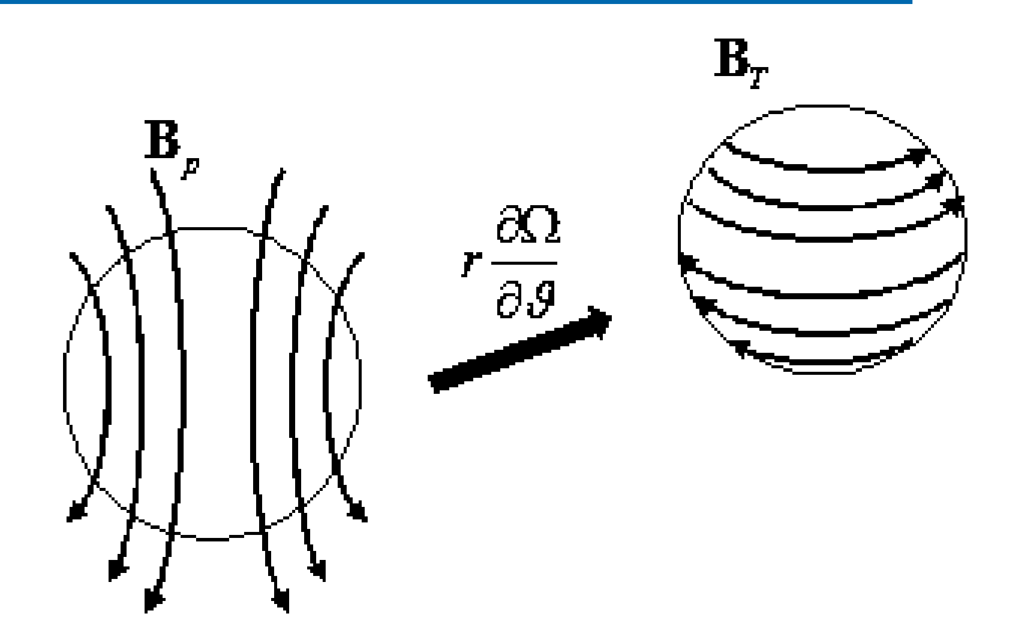
Kleeorin & Rogachevskii (2003)

Differential Rotation

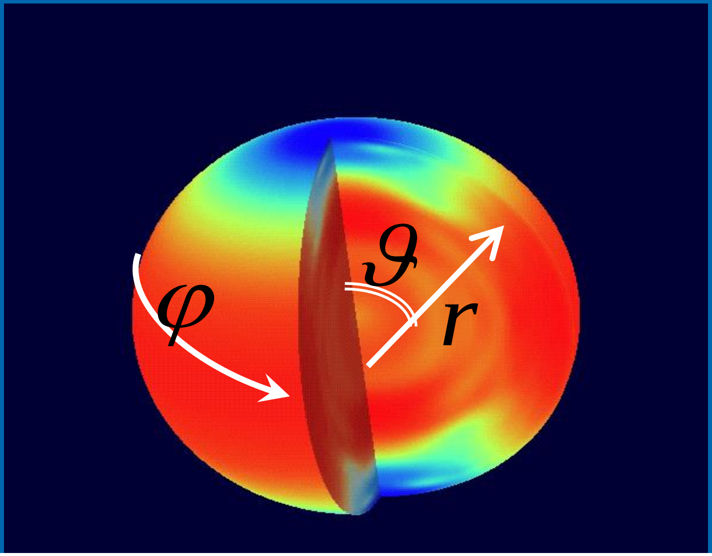
$$\mathbf{U} = \mathbf{e}_\varphi \Omega(r,\vartheta)\, r \sin(\vartheta)$$



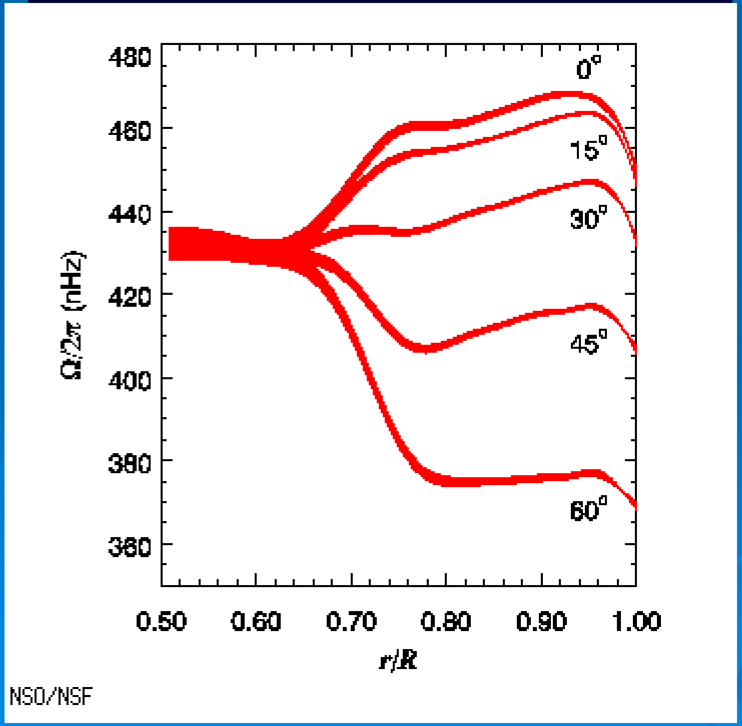


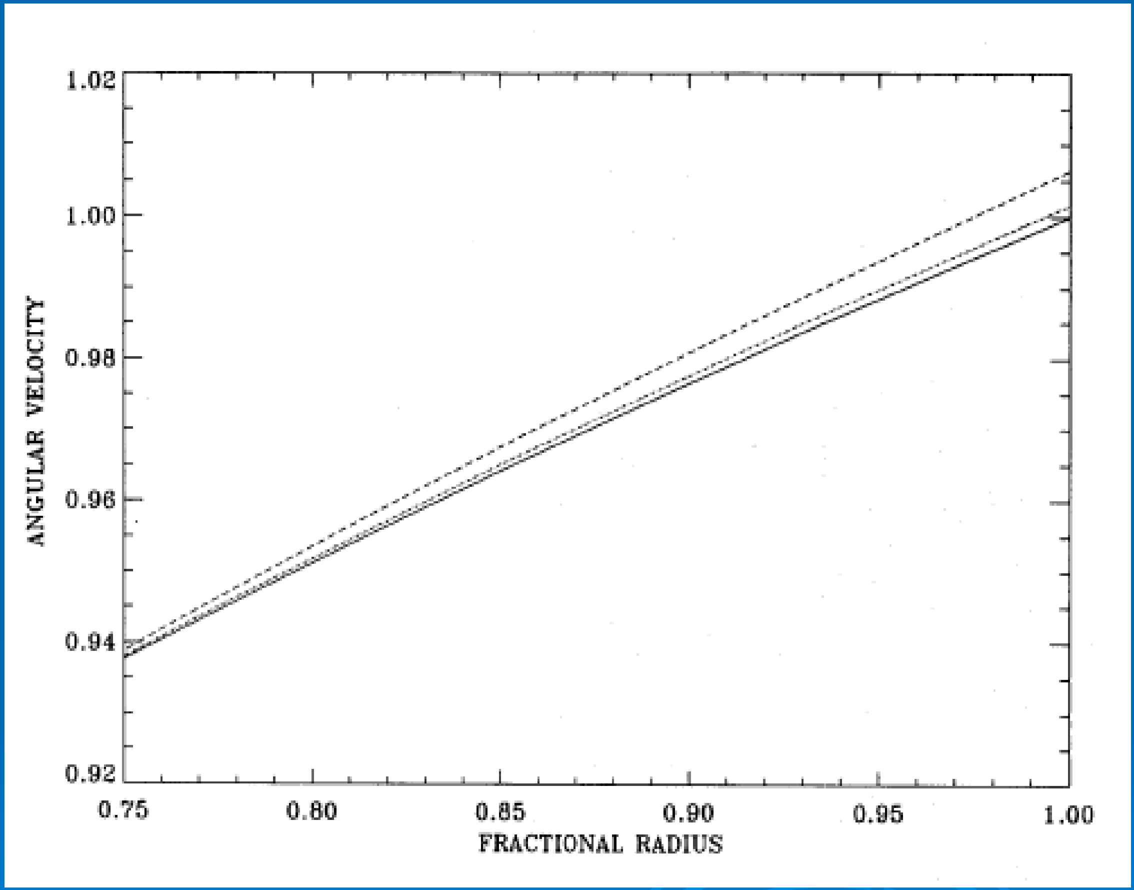


Differential Rotation



$$\mathbf{U} = \mathbf{e}_{\varphi}\Omega\left(r,\vartheta\right) r \sin(\vartheta)$$



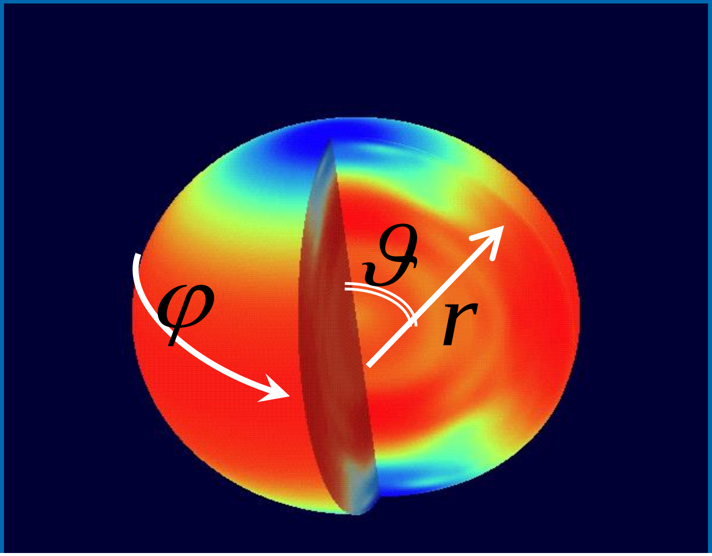


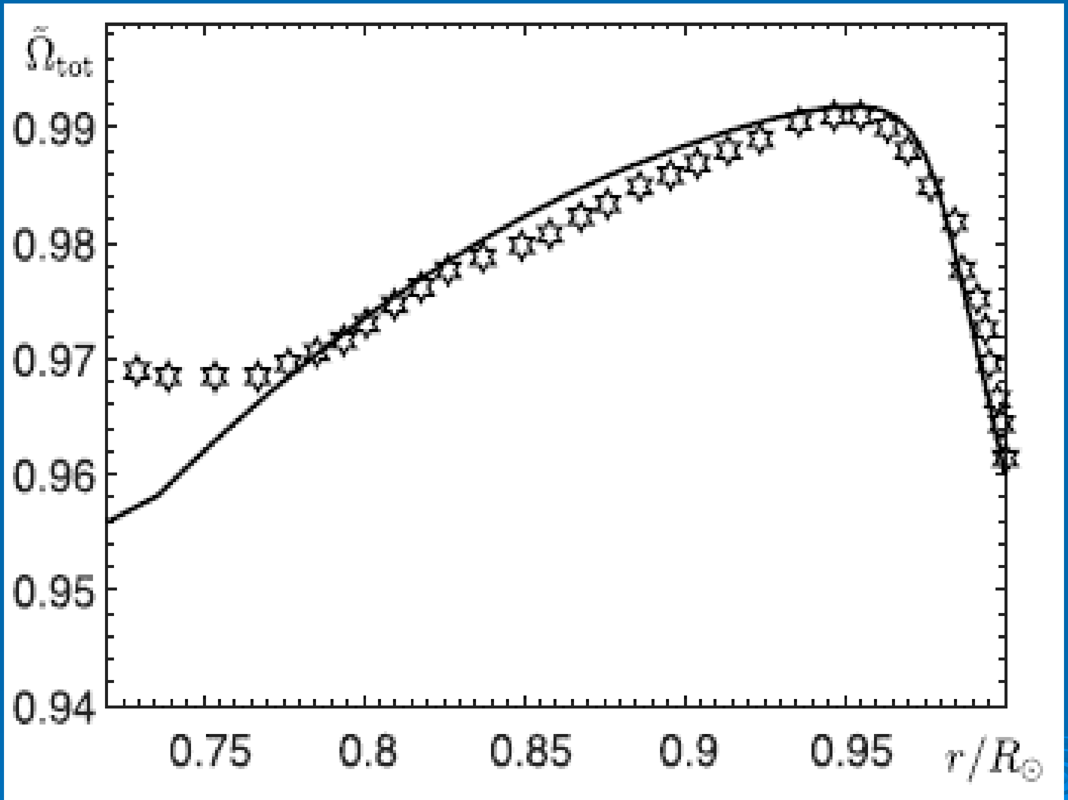
Kichatinov, L. L. & Rudiger, G. (1993)

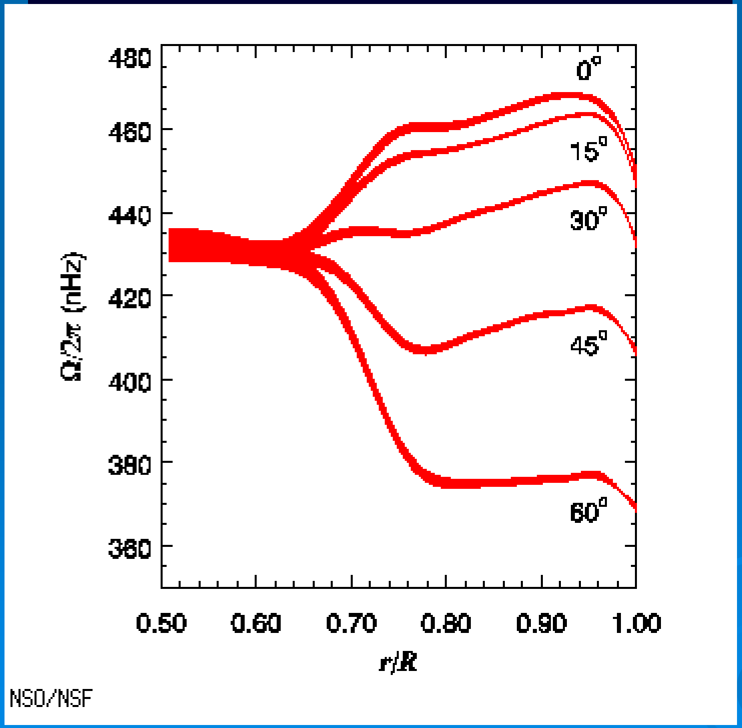
Differential Rotation

Крым 2019

$$\mathbf{U} = \mathbf{e}_{\varphi}\Omega(r,\vartheta)\, r\sin(\vartheta)$$

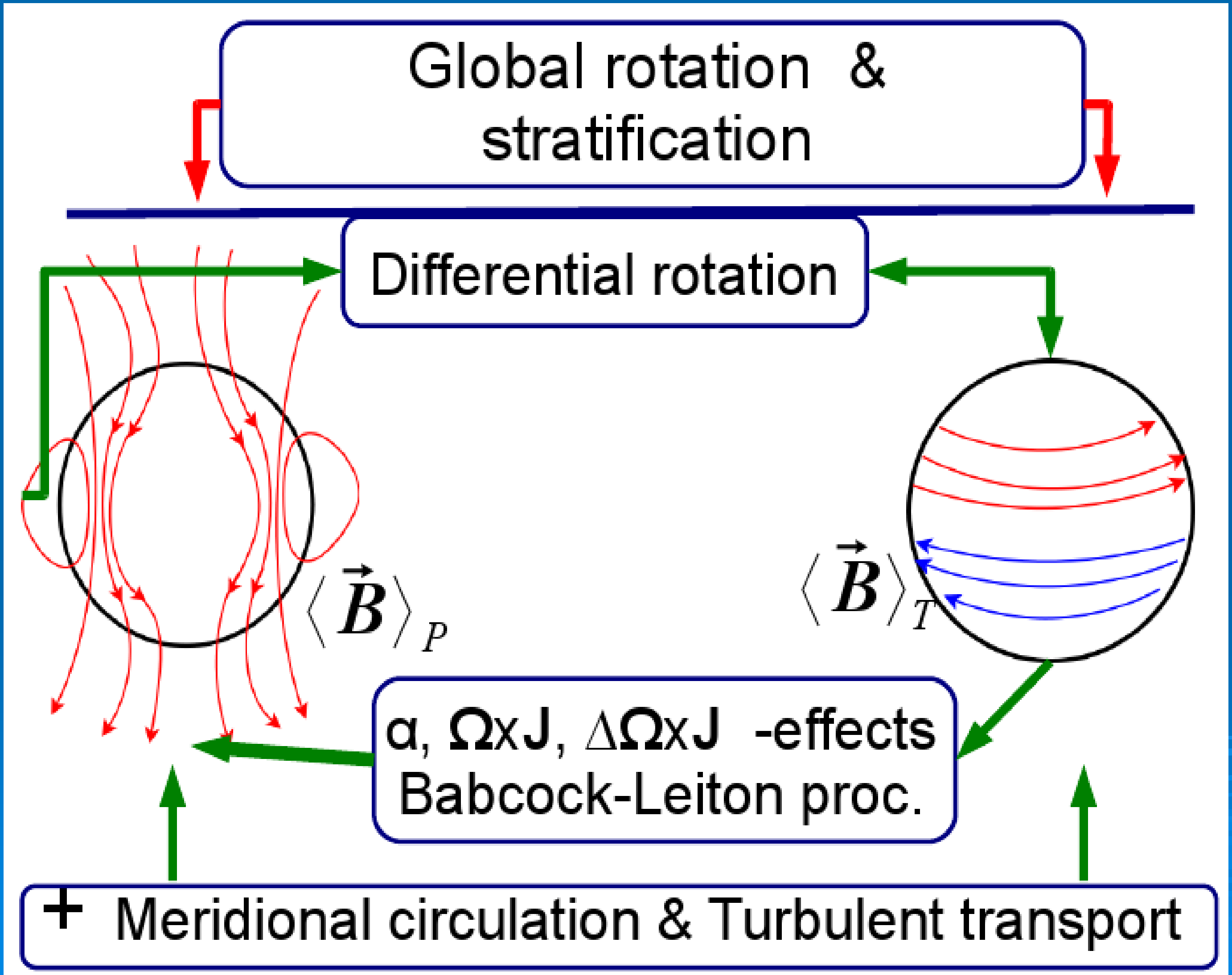






Rogachevskii & Kleeorin (2018)

Basic Mechanisms

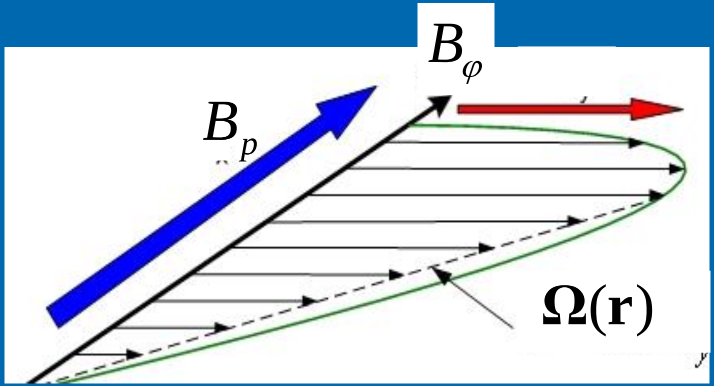


Generation of the mean magnetic field due to the $\alpha\Omega$ dynamo

Mean magnetic field:

$$\mathbf{B} = \mathbf{e}_\varphi B(r,\vartheta) + \mathrm{rot}\left[\mathbf{e}_\varphi A(r,\vartheta)\right]$$

$$r\sin(\vartheta)\left[\nabla\Omega\times\nabla A\right]_\varphi = \frac{1}{r}\left(\frac{\partial\Omega}{\partial r}\frac{\partial}{\partial\vartheta} - \frac{\partial\Omega}{\partial\vartheta}\frac{\partial}{\partial r}\right)(rA\sin(\vartheta))$$

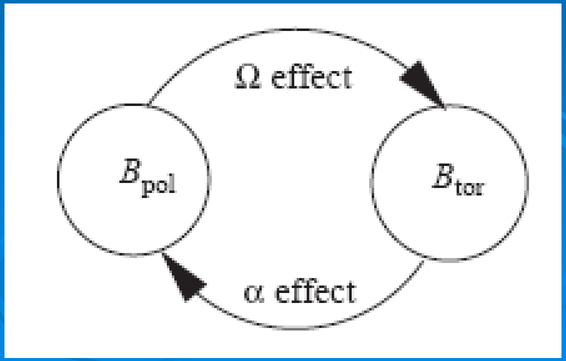


$$\frac{\partial B}{\partial t} = rD\sin(\vartheta)\left[\nabla\Omega\times\nabla A\right]_\varphi + \eta_T\Delta_s B - R_\alpha^2\,\hat{\Delta}(\alpha A)$$

$$\frac{\partial A}{\partial t} = \alpha B + \Delta_s A; \qquad \Delta_s = \Delta - (r\sin\vartheta)^{-2}$$

Dynamo number:

$$D = R_\alpha R_\Omega; \quad R_\alpha = \frac{\alpha R_\odot}{\eta_T}; R_\Omega = \frac{\delta\Omega R_\odot^2}{\eta_T};$$



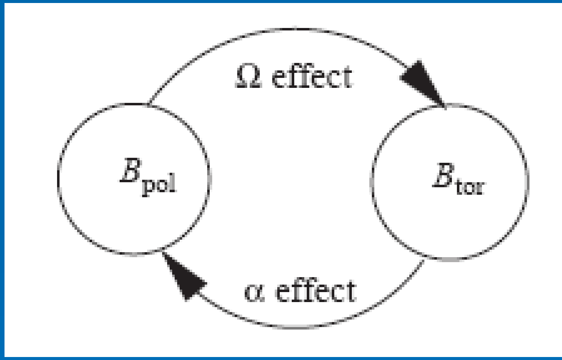
Generation of the mean magnetic field due to the $\alpha\Omega$ dynamo

Mean magnetic field:

$$\mathbf{B} = B_\varphi\, \mathbf{e}_\varphi + \nabla \times [A\, \mathbf{e}_\varphi]$$

$$\left(\frac{\partial}{\partial t} - \Delta\right) B = D \sin(\vartheta_0) \frac{\partial \Omega}{\partial r} \frac{\partial A}{\partial \vartheta} \Bigg| \left(\frac{\partial}{\partial t} - \eta_T \Delta\right)$$

$$\left(\frac{\partial}{\partial t} - \Delta\right) A = \cos(\vartheta_0) B$$



Dynamo number:

$$D = R_\alpha R_\Omega; \quad R_\alpha = \frac{\alpha_0 R_\odot}{\eta_T}; R_\Omega = \frac{\delta \Omega R_\odot^2}{\eta_T};$$

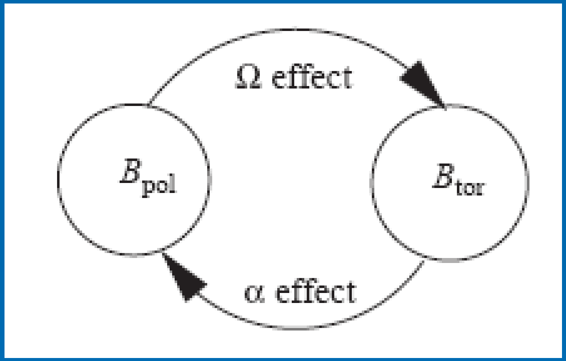
$$\Delta_s = \Delta - \frac{1}{r^2 \sin^2 \theta}$$

Generation of the mean magnetic field due to the $\alpha\Omega$ dynamo

Mean magnetic field:

$$\mathbf{B} = B_\varphi\,\mathbf{e}_\varphi + \nabla\times[A\,\mathbf{e}_\varphi]$$

$$B = B_0 \exp\left(\lambda t - i\left(K_r r + K_\vartheta \vartheta\right)\right)$$



$$\left(\frac{\partial}{\partial t} - \Delta\right)^2 B = D\cos(\vartheta_0)\sin(\vartheta_0)\frac{\partial\Omega}{\partial r}\frac{\partial B}{\partial\vartheta}$$

$$\left(\frac{\partial}{\partial t} - \Delta\right) A = \cos(\vartheta_0) B$$

Dynamo number:

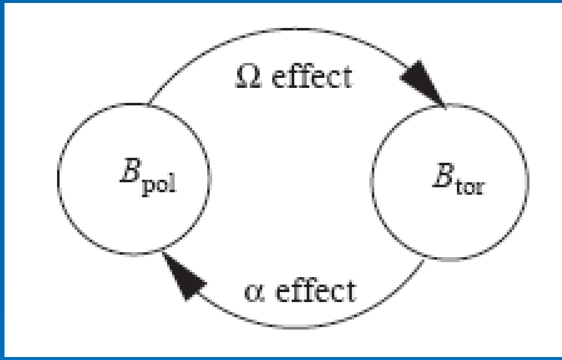
$$\Delta_s = \Delta - \frac{1}{r^2\sin^2\theta}$$

$$D = R_\alpha R_\Omega; \quad R_\alpha = \frac{\alpha_0 R_\odot}{\eta_T}; R_\Omega = \frac{\delta\Omega R_\odot^2}{\eta_T};$$

Generation of the mean magnetic field due to the $\alpha\Omega$ dynamo

Mean magnetic field: $B = B_0 \exp\left(\lambda t - i\left(K_r r + K_\vartheta \vartheta\right)\right) \Leftarrow$ **E. Parker (1955)**

$$\mathbf{B} = B_\varphi \, \mathbf{e}_\varphi + \nabla \times [A \, \mathbf{e}_\varphi]$$



$$\left(\lambda + \left(K_r^2 + K_\vartheta^2\right)\right)^2 B_0 = -B_0 \frac{iK_\vartheta}{2} D \sin(2\vartheta_0)$$

$$\left(\frac{\partial}{\partial t} - \Delta\right) A = \cos(\vartheta_0) B$$

Dynamo number:

$$D = R_\alpha R_\Omega; \quad R_\alpha = \frac{\alpha_0 R_\odot}{\eta_T}; R_\Omega = \frac{\delta\Omega R_\odot^2}{\eta_T};$$

$$\Delta_s = \Delta - \frac{1}{r^2 \sin^2 \theta}$$

Generation of the mean magnetic field due to the $\alpha\Omega$ dynamo

Mean magnetic field:

$$\mathbf{B} = B_\varphi \, \mathbf{e}_\varphi + \nabla \times [A \, \mathbf{e}_\varphi]$$

$$B = B_0 \exp(\gamma t)\cos\left(\omega t - i\left(K_r r + K_\vartheta \vartheta\right)\right)$$

$$\gamma = \frac{1}{2}\sqrt{\left|K_\vartheta \sin(2\vartheta_0) D\right|} - \left(K_r^2 + K_\vartheta^2\right); \quad \frac{d\gamma}{dK_\vartheta} = 0$$

$$K_0 = \frac{1}{2}\sqrt[3]{\frac{\left|\sin(2\vartheta_0) D\right|}{2}};$$

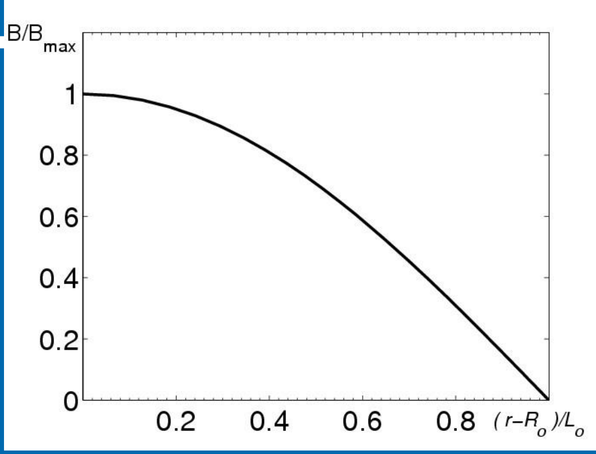
Dynamo number:

$$D = R_\alpha R_\Omega; \quad R_\alpha = \frac{\alpha_0 R_\odot}{\eta_T}; R_\Omega = \frac{\delta\Omega R_\odot^2}{\eta_T};$$

$$\gamma(K_0) = \left[\frac{1}{2}\left(\frac{\left|\sin(2\vartheta_0) D\right|}{2}\right)^{2/3} - \frac{1}{4}\left(\frac{\left|\sin(2\vartheta_0) D\right|}{2}\right)^{2/3} - K_r^2\right]; D = \alpha_0 \frac{\partial \Omega}{\partial r}\frac{R_\odot^4}{\eta_T^2} < 0$$

Generation of the mean magnetic field due to the $\alpha\Omega$ dynamo

$$B = B_0 \exp(\gamma t) \cos(\gamma t - K_0 \vartheta + \delta) \cos(K_r (1 - r))$$



Mean magnetic field:

$$\mathbf{B} = B_\varphi \mathbf{e}_\varphi + \nabla \times [A \mathbf{e}_\varphi]$$

$$K_0 = \frac{1}{2} \sqrt[3]{\frac{|\sin(2\vartheta_0) D|}{2}}; \qquad K_r^{(\min)} = R_\odot \frac{\pi}{2L_\odot}$$

$$\gamma(K_0) = \left[\frac{1}{4} \left(\frac{|\sin(2\vartheta_0) D|}{2} \right)^{2/3} - R_\odot^2 K_r^2 \right] < \left[\frac{1}{4} \left(\frac{|D|}{2} \right)^{2/3} - K_r^2 \right]$$

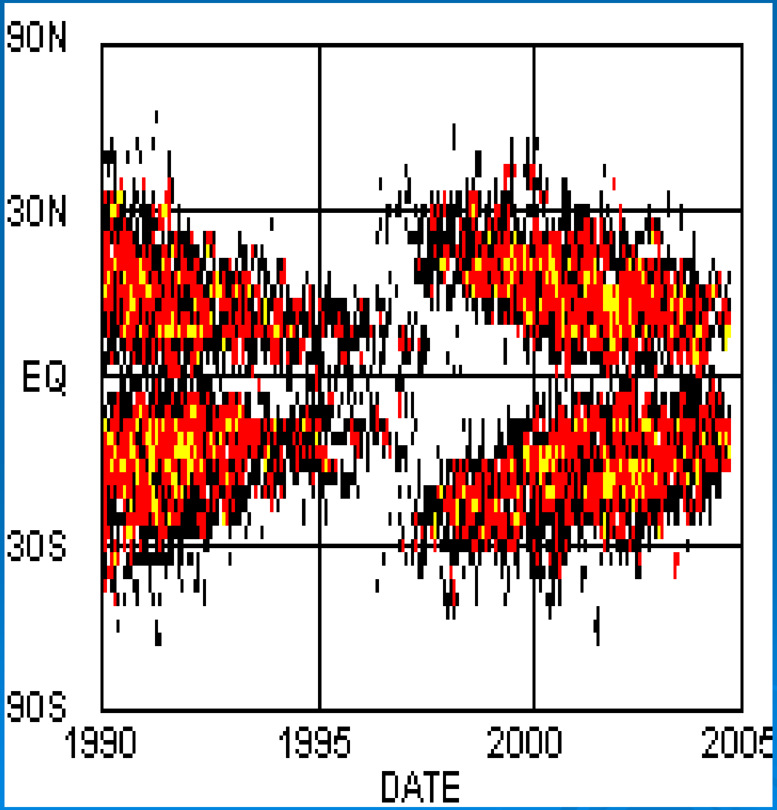
$$D = \alpha_0 \frac{\partial \Omega}{\partial r} \frac{R_\odot^4}{\eta_T^2} < 0; \ |D| > D_{cr} = 2(2K_r)^3 \approx 2(\pi R_\odot / L_\odot)^3 \approx \underline{1,680}$$

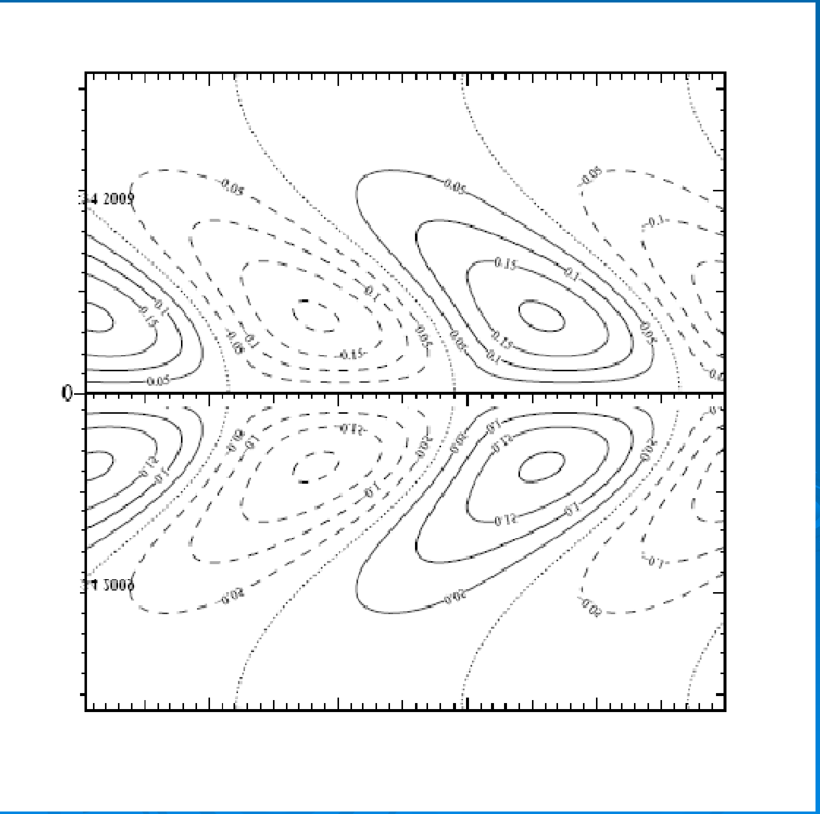
Comparing Observations and Theory

$$B_{00} - ?$$

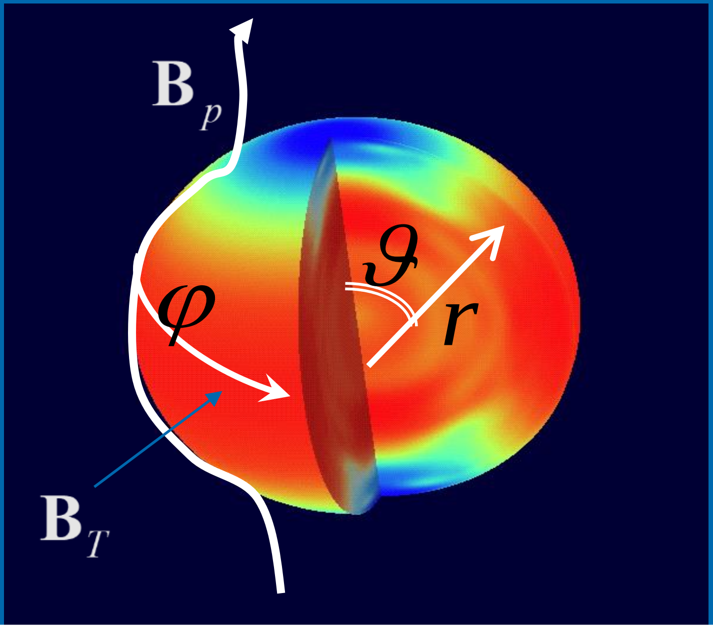
$$B = B_{00}\exp(\gamma t)\cos(\gamma t - K_0\vartheta + \delta)\cos(K_r(1-r))$$

Steenbeck, Krause and Raedler (1966)





Nonlinear Alpha-Omega Dynamo (Mean-Field Approach)



$$\frac{\partial}{\partial\varphi} = 0$$

$$\mathbf{U} = \mathbf{e}_\varphi \Omega(r,\vartheta)\, r\sin(\vartheta);$$

Iroshnikov (1970)

$$\alpha(r,\vartheta,B) = \frac{\alpha(r,\vartheta)}{1+\left(B/B_0\right)^2}$$

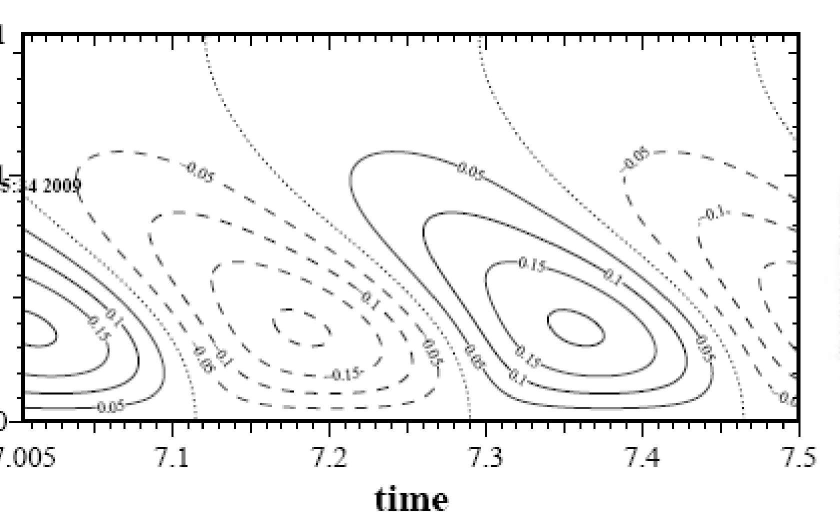
$$\frac{\partial B}{\partial t} = \left(\mathbf{B}_p \cdot \nabla\right)\mathbf{U}_T + \eta_T \Delta_s B$$

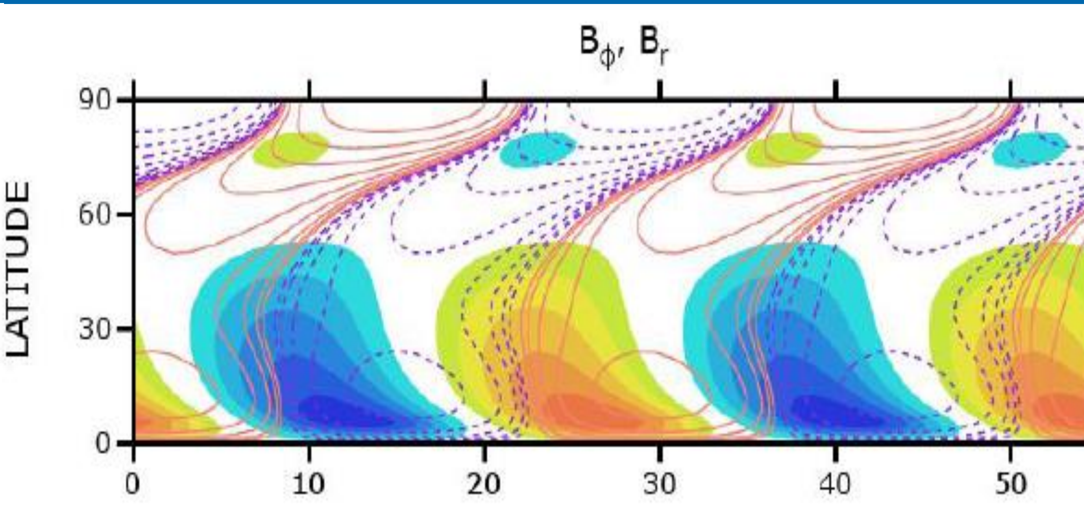
$$\frac{\partial A}{\partial t} = \alpha(r,\vartheta,B)\, B + \eta_T \Delta_s A; \; \Delta_s = \Delta - \left(r\sin\vartheta\right)^{-2}$$

$B_0 - ?$

Nonlinear Alpha-Omega Dynamo (Mean-Field Approach)

Steenbeck, Krause and Raedler (1966) **~ e.g., Pipin (2000)**





$$\frac{\partial B}{\partial t} = \left(\mathbf{B}_p \cdot \nabla\right)\mathbf{U}_T + \eta_T \Delta_s B \qquad \alpha\left(r, \vartheta, B\right) = \frac{\alpha\left(r, \vartheta\right)}{1 + \left(B/B_0\right)^2}$$

Iroshnikov (1970)

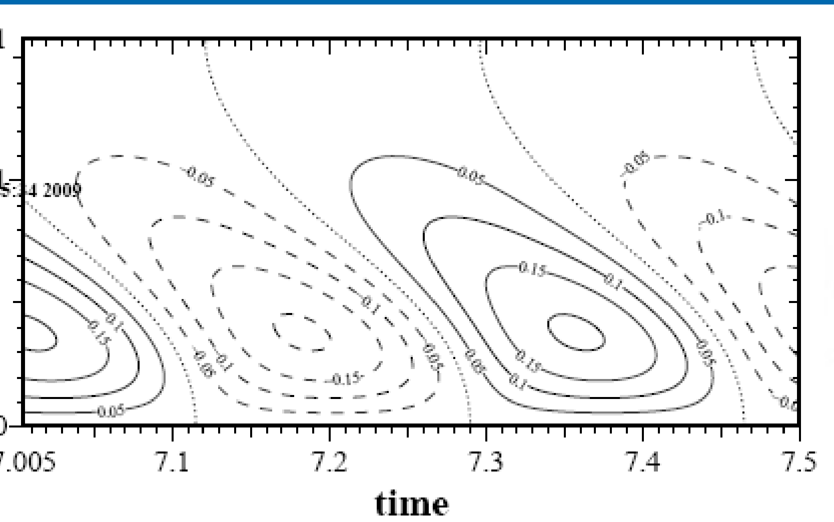
$$\frac{\partial A}{\partial t} = \alpha\left(r, \vartheta, B\right) B + \eta_T \Delta_s A; \; \Delta_s = \Delta - \left(r \sin\vartheta\right)^{-2}$$

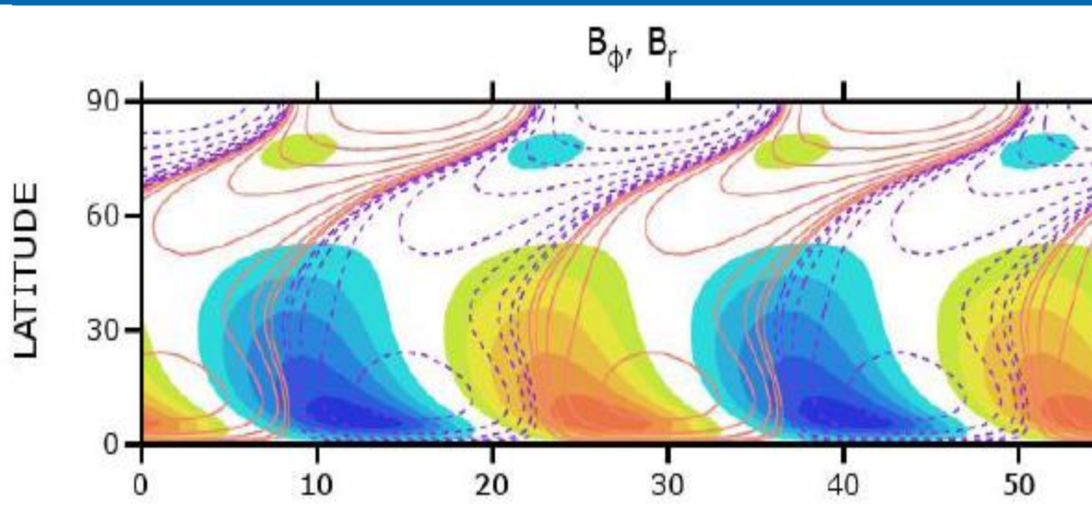
$B_{00} - ?$

Nonlinear Alpha-Omega Dynamo (Mean-Field Approach)

Steenbeck, Krause and Raedler (1966)

Pipin (2000)





$$\frac{\partial B}{\partial t} = \left(\mathbf{B}_p \cdot \nabla\right)\mathbf{U}_T + \eta_T \Delta_s B \rightarrow$$

$$\rightarrow D(B) = \frac{D}{1+\left(B/B_0\right)^2} \rightarrow D_{cr}$$

$$\frac{\partial A}{\partial t} = \alpha\left(r,\vartheta,B\right)B + \eta_T \Delta_s A;$$

$$\Delta_s = \Delta - \left(r \sin\vartheta\right)^{-2}$$

Iroshnikov (1970)

$$\alpha\left(r,\vartheta,B\right) = \frac{\alpha\left(r,\vartheta\right)}{1+\left(B/B_0\right)^2} \rightarrow$$

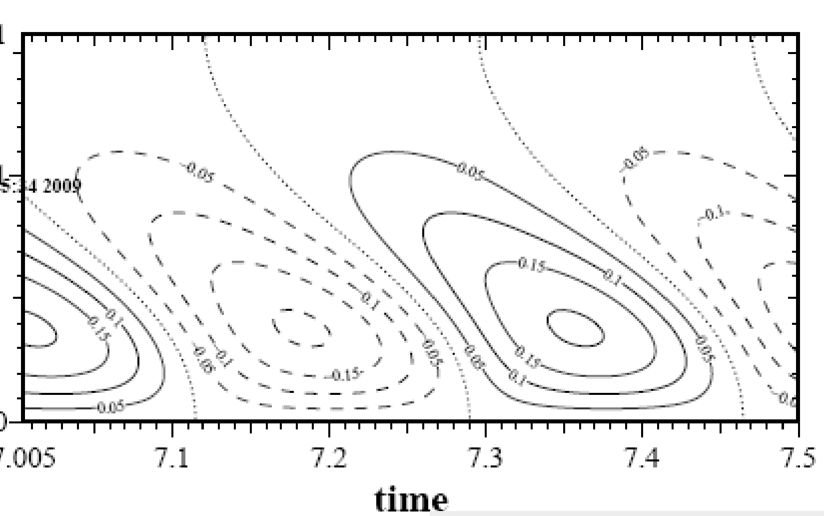
$$\rightarrow B \approx B_0 \sqrt{\frac{D}{D_{cr}} - 1}$$

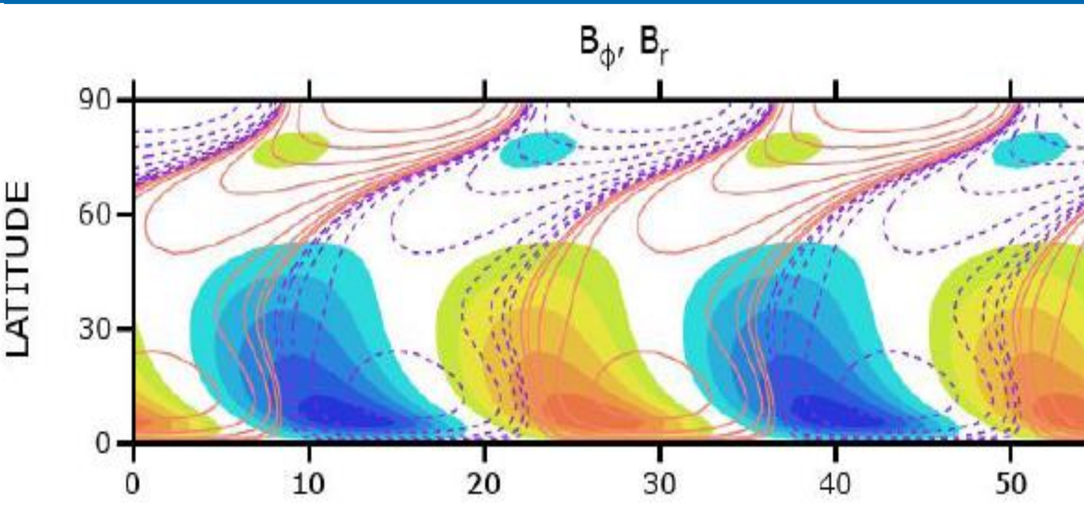
$$B_{00} - ?$$

Nonlinear Alpha-Omega Dynamo (Mean-Field Approach)

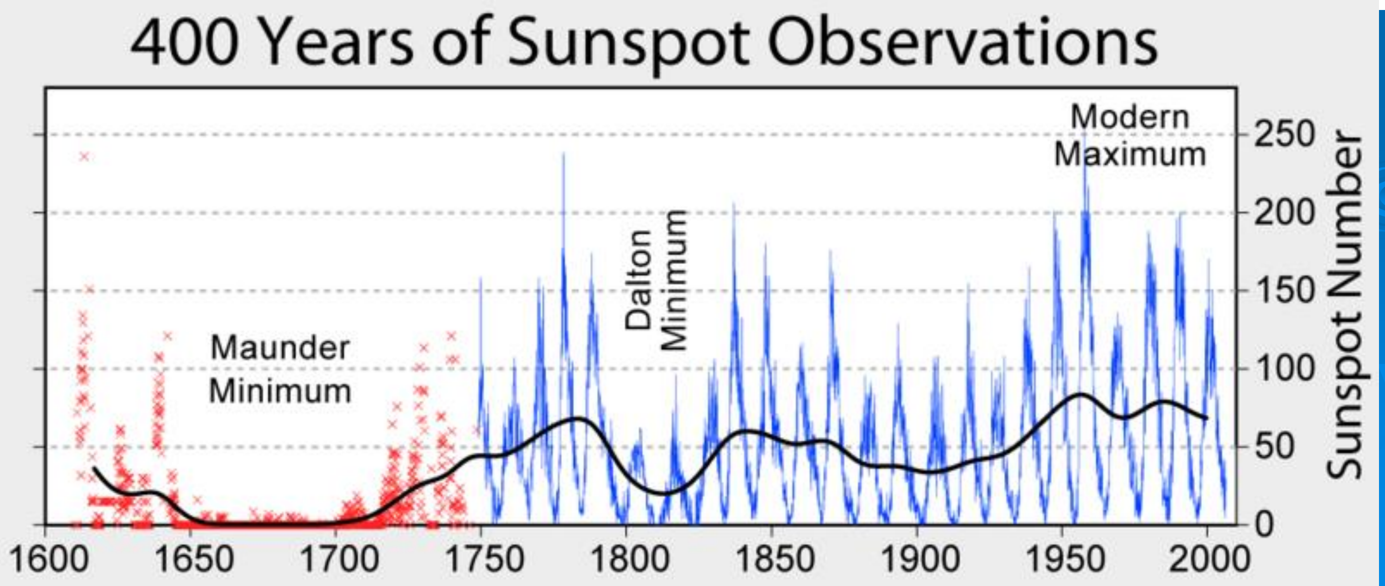
Steenbeck, Krause and Raedler (1966)

Pipin (2000)





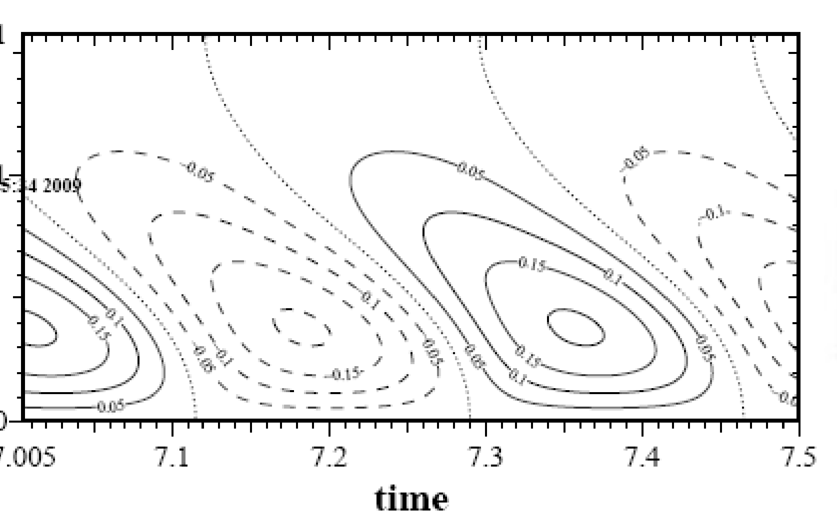
$$W \approx 11 N_{Sp}$$

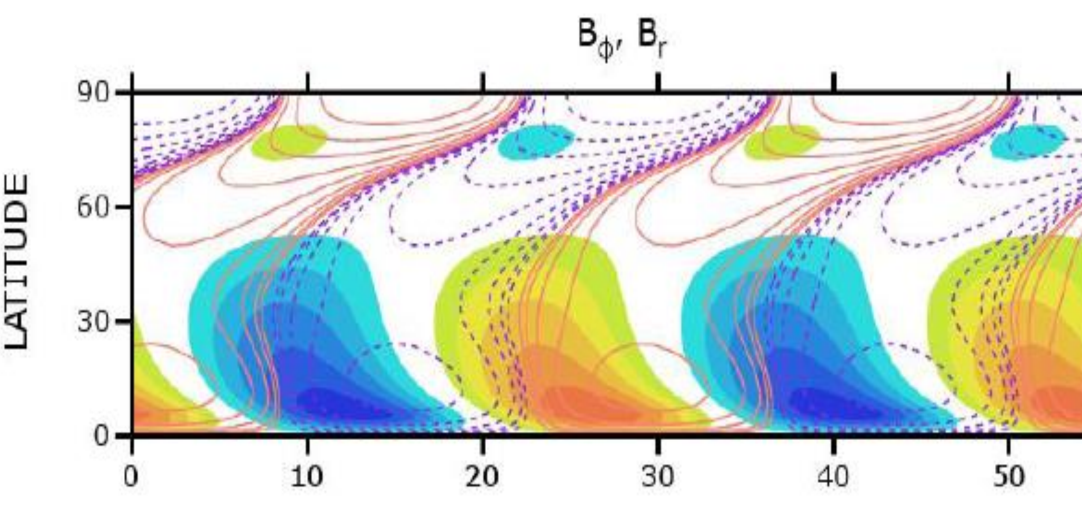


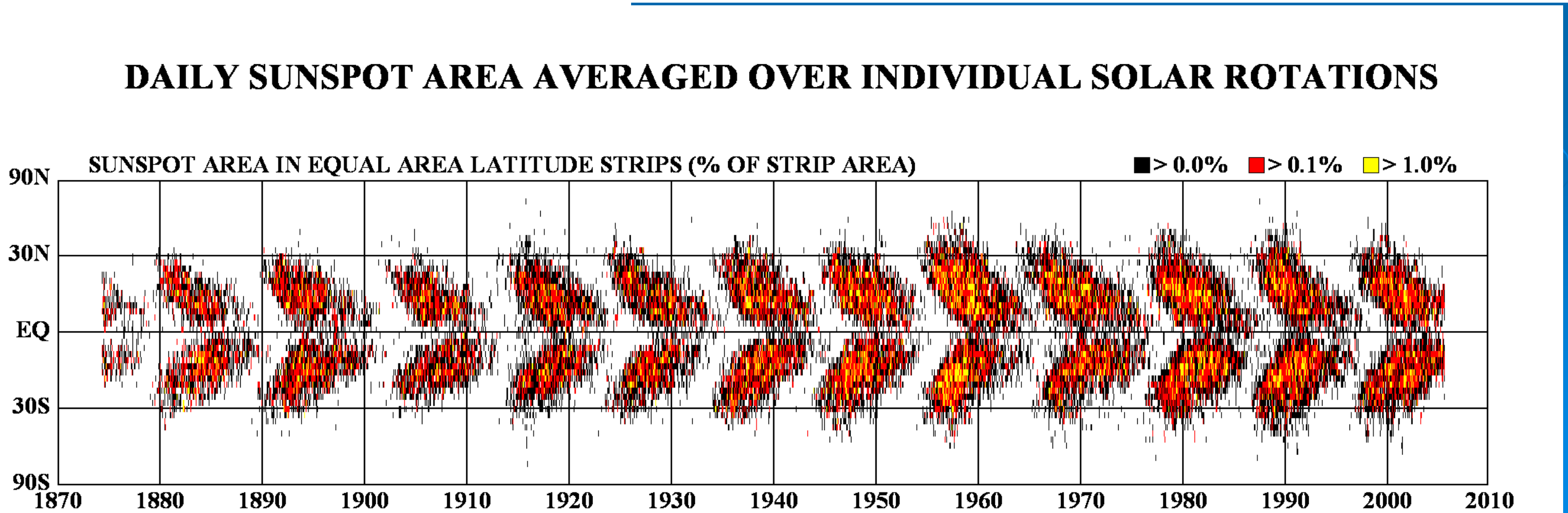
B_{00} – ? Nonlinear Alpha-Omega Dynamo (Mean-Field Approach)

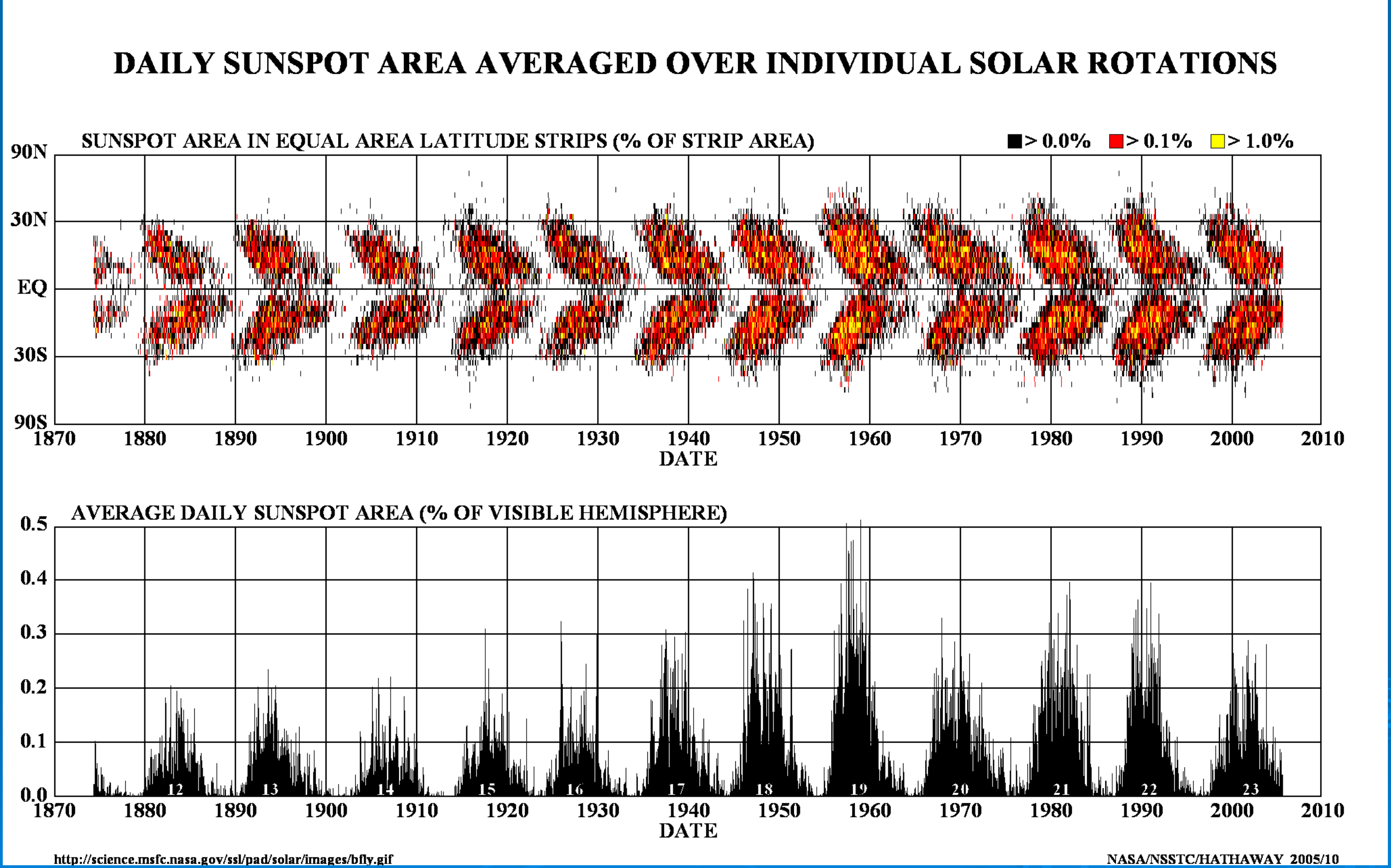
Steenbek, Krause and Redler (1966)

Pipin (2000)








DAILY SUNSPOT AREA AVERAGED OVER INDIVIDUAL SOLAR ROTATIONS
SUNSPOT AREA IN EQUAL AREA LATITUDE STRIPS (% OF STRIP AREA)
■> 0.0% ■> 0.1% □> 1.0%
90N
30N
EQ
30S
90S
1870 1880 1890 1900 1910 1920 1930 1940 1950 1960 1970 1980 1990 2000 2010
DATE
AVERAGE DAILY SUNSPOT AREA (% OF VISIBLE HEMISPHERE)
0.5
0.4
0.3
0.2
0.1
0.0
12 13 14 15 16 17 18 19 20 21 22 23
1870 1880 1890 1900 1910 1920 1930 1940 1950 1960 1970 1980 1990 2000 2010
DATE
http://science.msfc.nasa.gov/ssl/pad/solar/images/bfly.gif
NASA/NSSTC/HATHAWAY 2005/10

Nonlinear Dynamo

Magnetic part of Alpha -Effect (Mean-Field Approach)

- **Induction equation for mean magnetic field:**

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times \left(\mathbf{U} \times \mathbf{B} + \boldsymbol{\varepsilon} - \eta \nabla \times \mathbf{B} \right)$$

- **Electromotive force:**

$$\boldsymbol{\varepsilon} \equiv \left\langle \mathbf{u} \times \mathbf{b} \right\rangle = \boldsymbol{\alpha} \mathbf{B} - \boldsymbol{\eta}_{\mathbf{T}} \nabla \times \mathbf{B} + \ldots$$

$$\boldsymbol{\alpha} = -\frac{\tau}{3} \left\langle \mathbf{u} \cdot \mathbf{rot}\, \mathbf{u} \right\rangle + \frac{\tau}{12\pi\rho} \underbrace{\left\langle \mathbf{b} \cdot \mathbf{rot}\, \mathbf{b} \right\rangle}_{\sim \mathbf{a} \bullet \mathbf{b}}$$

Evolution equation magnetic for part of Alpha -Effect

$$\frac{\partial \alpha_m}{\partial t} + \mathrm{div}\, \mathbf{F} = \frac{Q}{4\pi\rho\eta_T}\left[-\left(\alpha_h + \alpha_m\right)\mathbf{B}^2 + \eta_T \mathbf{B}\cdot \mathrm{rot}\mathbf{B}\right] - \frac{\alpha_m}{\tau_\chi}$$

$\tau_\chi \approx \tau_0 \mathrm{Rm}$ **Kleeorin & Ruzmaikin (1982)**

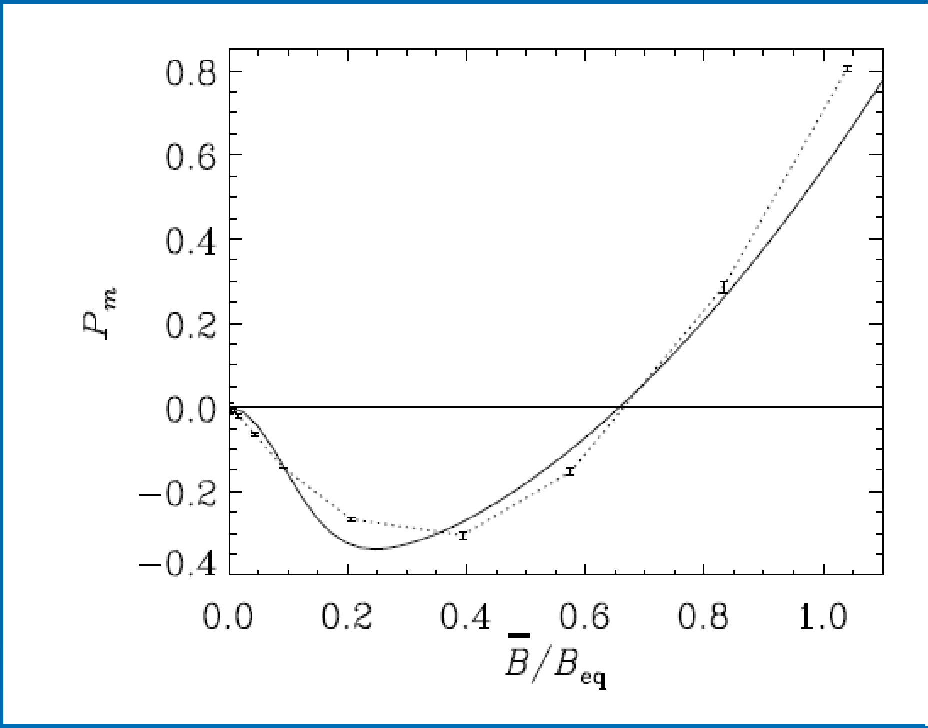
- **Electromotive force:**

$$\boldsymbol{\varepsilon} \equiv \langle \mathbf{u}\times\mathbf{b}\rangle = \boldsymbol{\alpha}\,\mathbf{B} - \boldsymbol{\eta}_\mathbf{T} \nabla\times\mathbf{B} + \ldots$$

$$\alpha = \overbrace{-\frac{\tau}{3}\langle \mathbf{u}\cdot\mathbf{rot}\ \mathbf{u}\rangle}^{\alpha_h} \varphi_k(B) + \overbrace{\frac{\tau}{12\pi\rho}\underbrace{\langle \mathbf{b}\cdot\mathbf{rot}\ \mathbf{b}\rangle}_{\sim \mathbf{a}\cdot\mathbf{b}}}^{\alpha_m} \varphi_m(B)$$

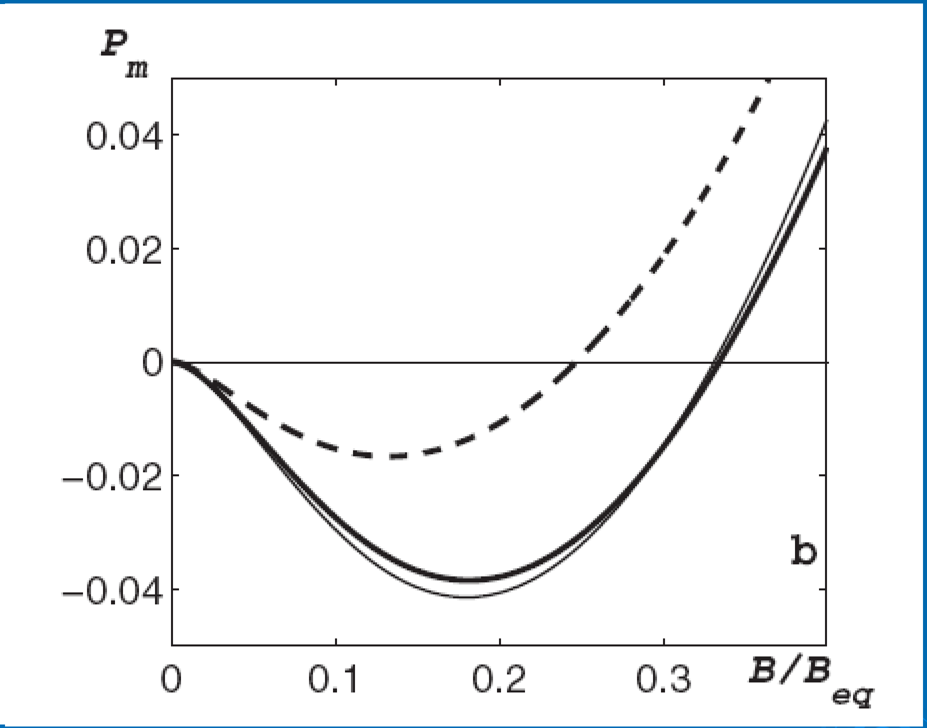
Field & Blackman (2000)

Negative Magnetic Pressure



(Brandenburg, Kleeorin, Rogachevskii 2010)

$$P_m = (1 - q_p)\frac{\overline{B}^2}{\overline{B}_{eq}^2}$$



(Kleeorin, Rogachvskii, Ruzmaikin 1989,1990; Rogachvskii, Kleeorin 2007)

Negative Magnetic Pressure

МОСКОВСКИЙ ОРДЕНА ЛЕНИНА И ОРДЕНА ТРУДОВОГО КРАСНОГО ЗНАМЕНИ
ГОСУДАРСТВЕННЫЙ ПЕДАГОГИЧЕСКИЙ ИНСТИТУТ имени В.И.ЛЕНИНА

КЛИОРИН НАТАН ИОСИФОВИЧ

УДК 524.3-337+523.9-337

НЕЛИНЕЙНОЕ ДИНАМО КРУПНОМАСШТАБНЫХ МАГНИТНЫХ
ПОЛЕЙ ЗВЕЗД

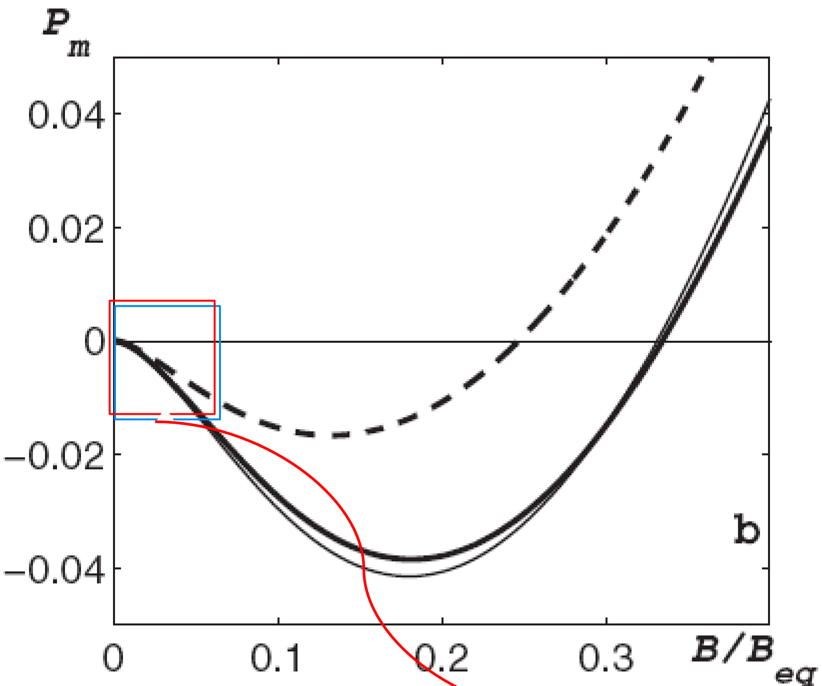
(Специальность - 0I.03.02 - астрофизика)

Диссертация на соискание
ученой степени кандидата
физико-математических наук

Научный руководитель:
доктор физико-математических
наук, старший научный сотрудник
А.А.РУЗМАЙКИН

Москва - I984

Negative Magnetic Pressure



$$\sigma^{M}_{ij} = \frac{1}{4\pi}\left[-\frac{K_1}{2}B^2\delta_{ij} + K B_i B_j + \frac{\langle h_0^2\rangle}{6}\delta_{ij}\right],$$

где $K_1 \simeq \left(1 - 2/15 \ln R_m\right)$,

$$K \simeq \left(1 - 2/45 \ln R_m\right). \quad \text{(I.3.II)}$$

How to Understand Negative Magnetic Pressure?

$$P_B = \frac{\overline{B}^2}{8\pi}; \quad p_T = \frac{1}{3}E_M + \frac{2}{3}E_K$$

$$p_T = -\frac{1}{24\pi}\left\langle b^2 \right\rangle + \frac{2}{3}E_T^{(tot)}; \quad E_T^{(tot)} = \frac{\rho_0 \left\langle u^2 \right\rangle}{2} + \frac{\left\langle b^2 \right\rangle}{8\pi}$$

$$P_{MHD} = P^{(tot)} - P_g = \frac{1}{8\pi}\left(\overline{B}^2 - \frac{\left\langle b^2 \right\rangle}{3} \right) + \frac{2}{3}E_T^{(tot)};$$

$$\left\langle b^2 \right\rangle = a\overline{B}^2; \quad E_T^{(tot)} = E_T^{(tot)}(0) - \frac{a_1}{8\pi}\overline{B}^2;$$

$$P_{MHD} = \frac{\overline{B}^2}{8\pi}\left(1 - \frac{a + 2a_1}{3} \right) + \frac{2}{3}E_T^{(tot)}(0)$$

How to Understand Negative Magnetic Pressure?

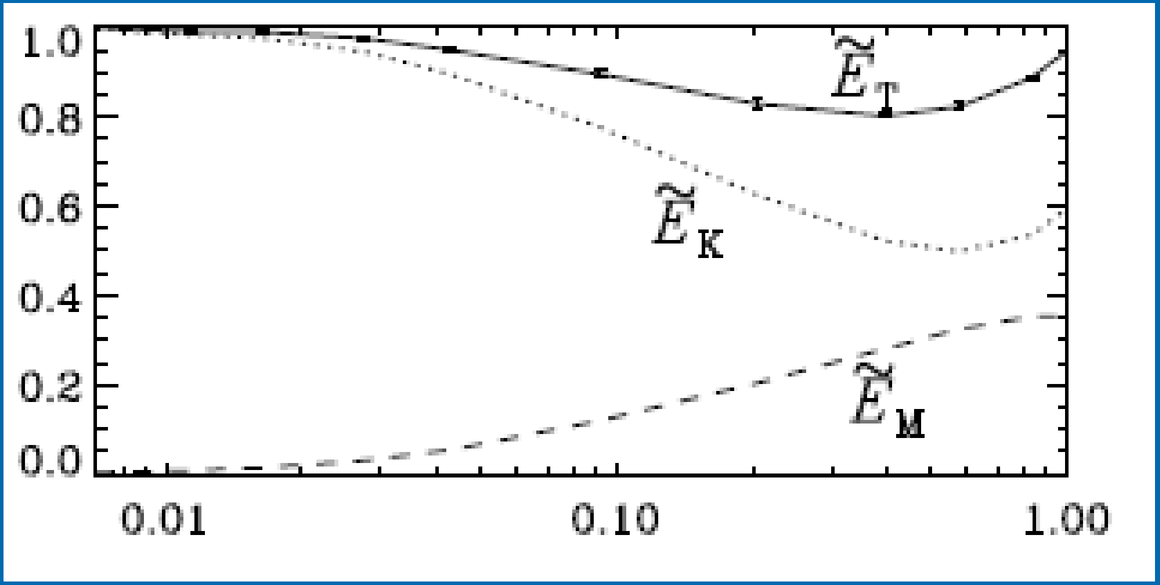
$$P_B = \frac{\overline{B}^2}{8\pi};$$

$$P_B^{eff} = \frac{\overline{B}^2}{8\pi}\left(1 - \frac{a + 2a_1}{3}\right)$$

$$\left\langle b^2 \right\rangle = a\overline{B}^2;$$

$$E_T^{(tot)} = E_T^{(tot)}(0) - \frac{a_1}{8\pi}\overline{B}^2;$$

$$P_{MHD} = \frac{\overline{B}^2}{8\pi}\left(1 - \frac{a + 2a_1}{3}\right) + \frac{2}{3}E_T^{(tot)}(0)$$

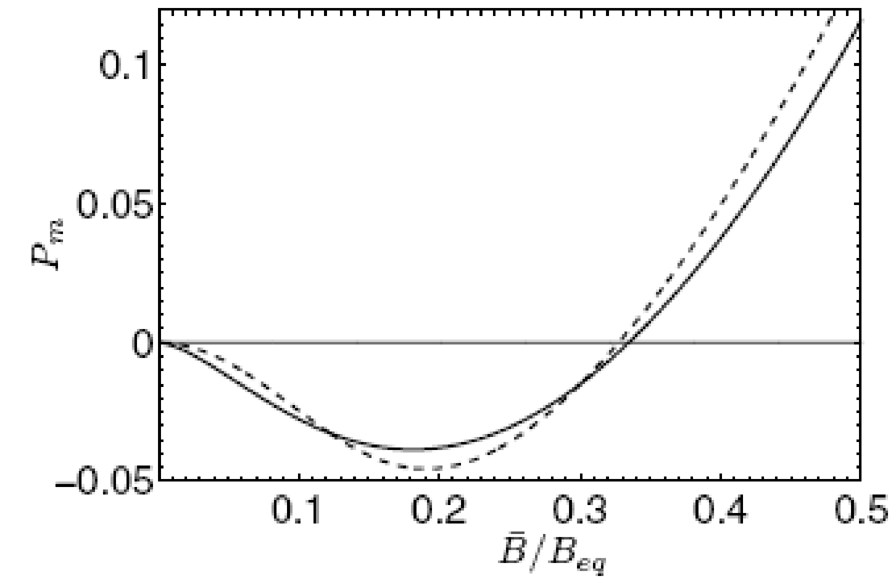


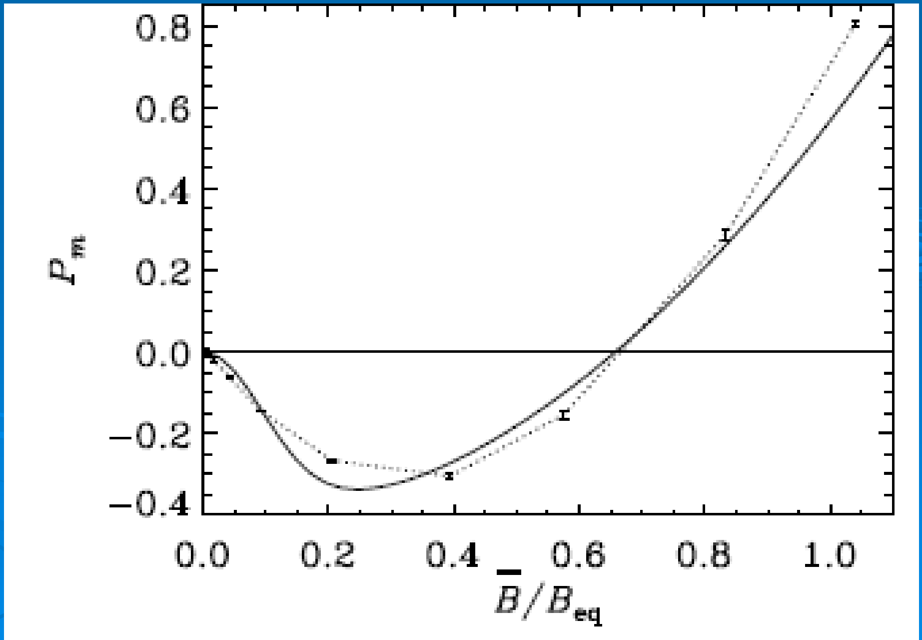
(Brandenburg, Kleeorin, Rogachevskii 2010)

Negative Magnetic Pressure

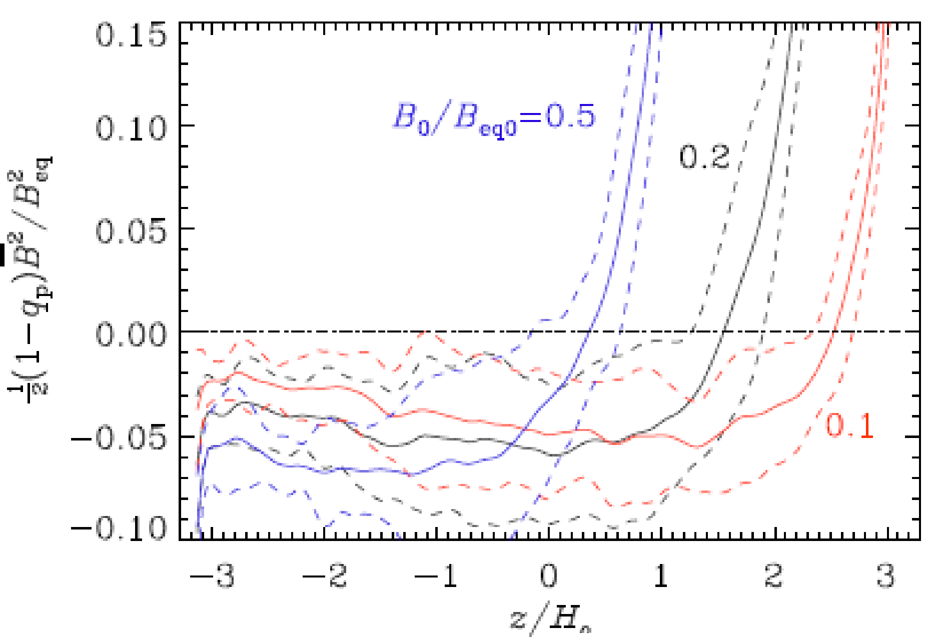
$$P_m = (1 - q_p)\frac{\overline{B}^2}{\overline{B}_{eq}^2}$$

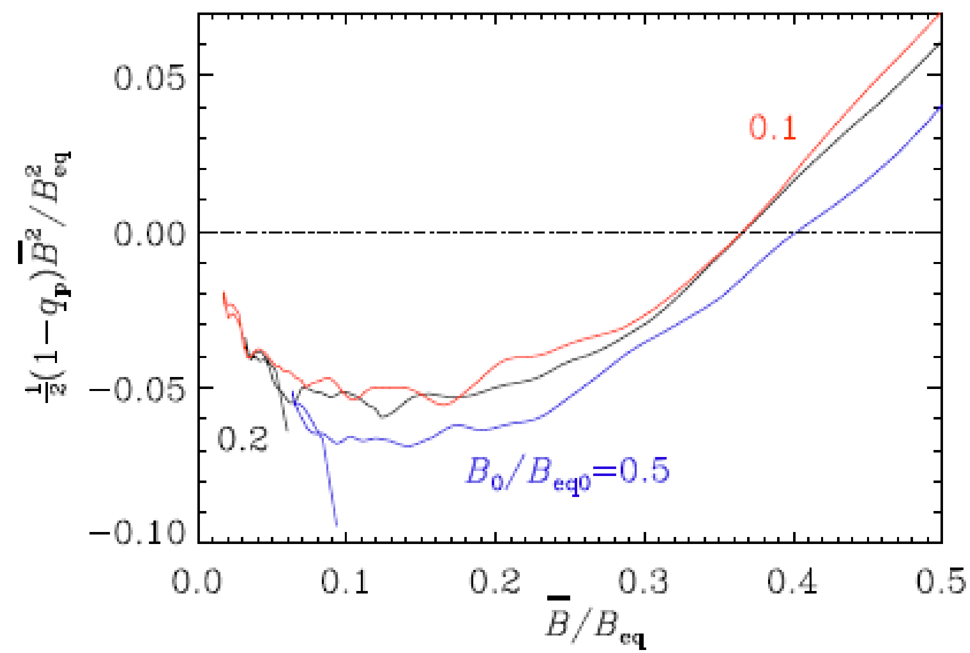
(Brandenburg, Kleeorin, Rogachevskii 2010)





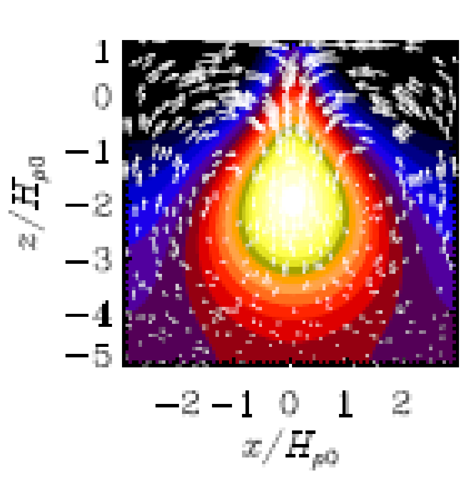
Negative Magnetic Pressure

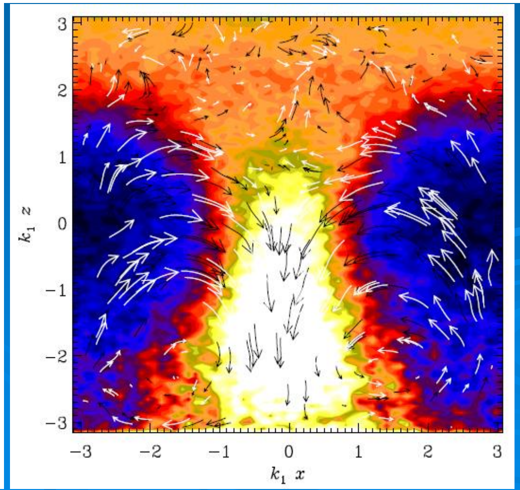




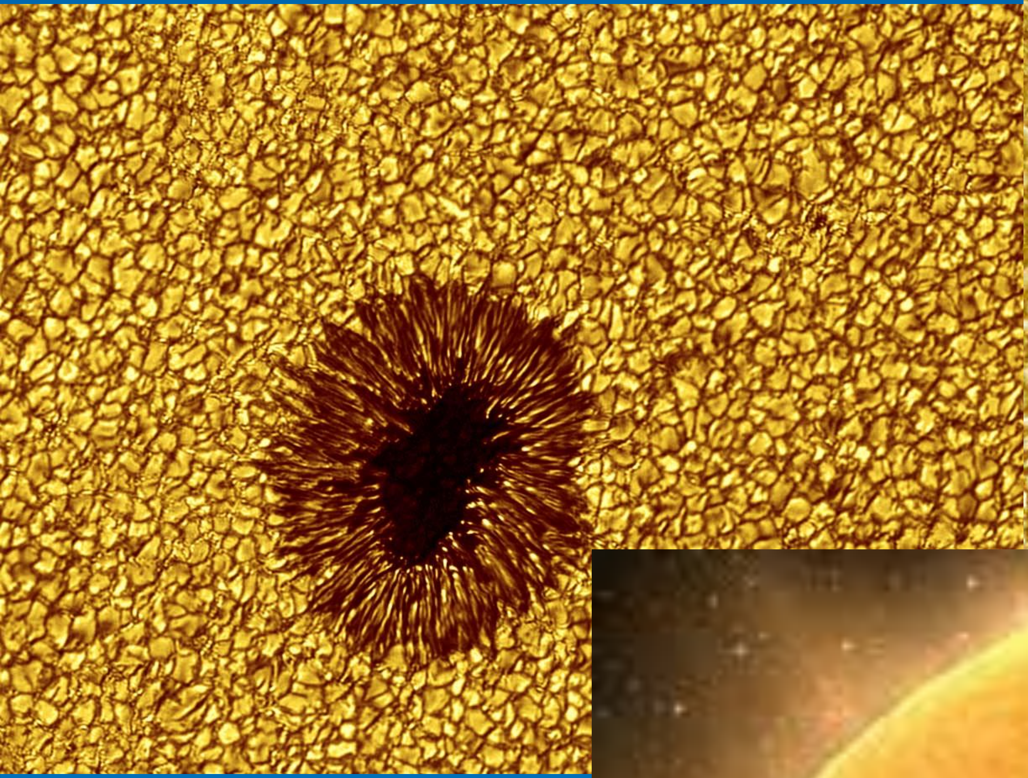
$$P_m = (1 - q_p)\frac{\overline{B}^2}{\overline{B}_{eq}^2}$$

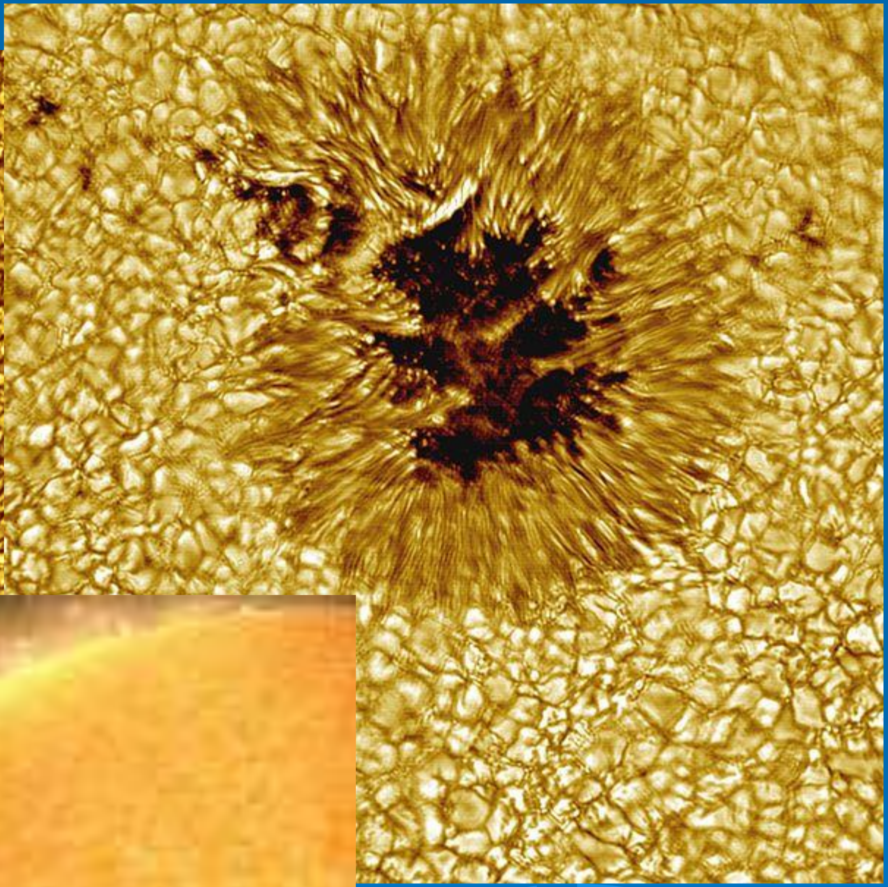
(Brandenburg & ell 2010-17)





Solar convection-examples

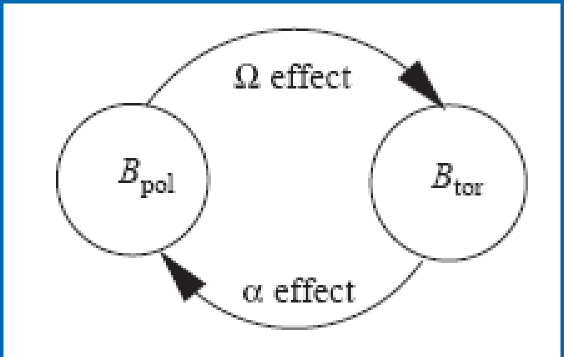




Nonlinear $\alpha\Omega$ dynamo

Mean magnetic field:

$$\mathbf{B} = \mathbf{e}_\varphi B(r,\vartheta) + \mathrm{rot}\left[\mathbf{e}_\varphi A(r,\vartheta)\right]$$



$$r\sin(\vartheta)\left[\nabla\Omega\times\nabla A\right]_\varphi = \frac{1}{r}\left(\frac{\partial\Omega}{\partial r}\frac{\partial}{\partial\vartheta} - \frac{\partial\Omega}{\partial\vartheta}\frac{\partial}{\partial r}\right)(rA\sin(\vartheta))$$

$$\frac{\partial B}{\partial t} = rD\sin(\vartheta)\left[\nabla\Omega\times\nabla A\right]_\varphi + \Delta_s B$$

$$\frac{\partial A}{\partial t} = \alpha B + \Delta_s A; \qquad \Delta_s = \Delta - (r\sin\vartheta)^{-2}$$

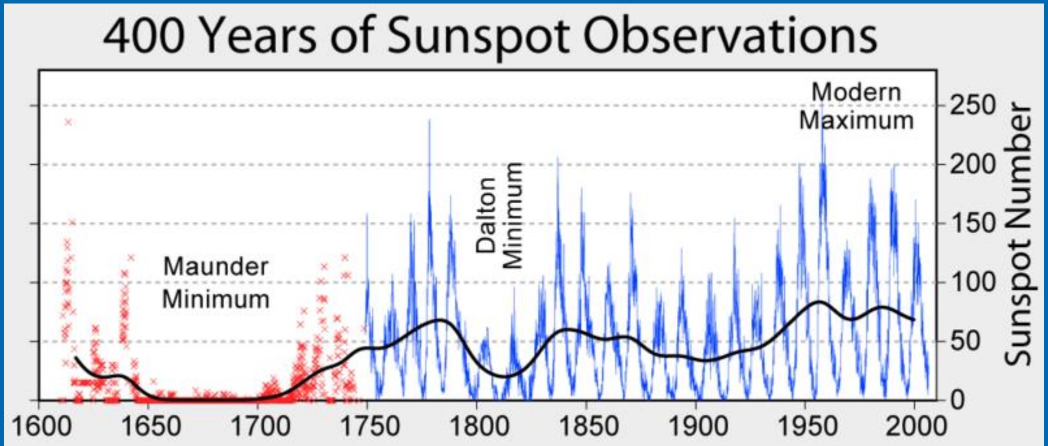
$$\frac{\partial\alpha_m}{\partial t} + \mathrm{div}\,\mathbf{F} = \frac{Q}{4\pi\rho\eta_T}\left[-(\alpha_h+\alpha_m)\mathbf{B}^2 + \eta_T\mathbf{B}\cdot\mathrm{rot}\mathbf{B}\right] - \frac{\alpha_m}{\tau_\chi}$$

$$\tau_\chi \approx \tau_0 \mathrm{Rm}$$

Kleeorin & Ruzmaikin (1982)

Reza Tavakol & Enrique Covas (1997)

RESULTS



$\phi_m =$

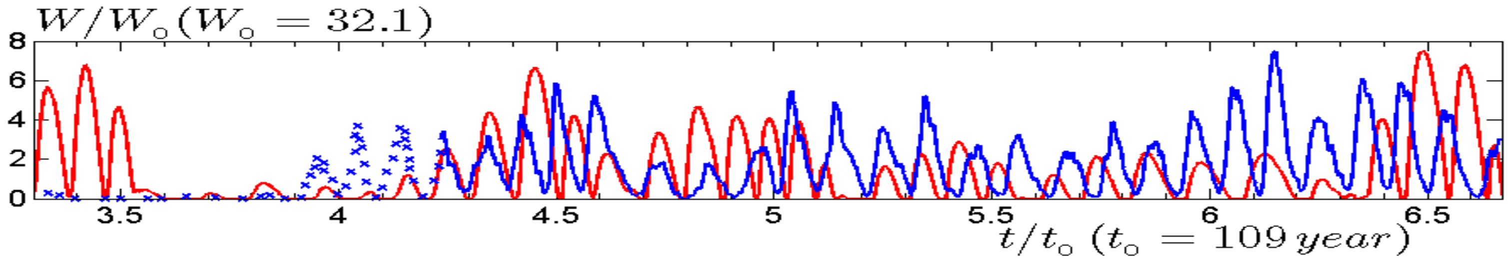
$3\left[1-\text{arctan}(\xi B_{tot})/\xi B_{tot}\right]/\xi^2 B_{tot}^2$;

$D = -\left(4 \div 10\right) \times 10^3$

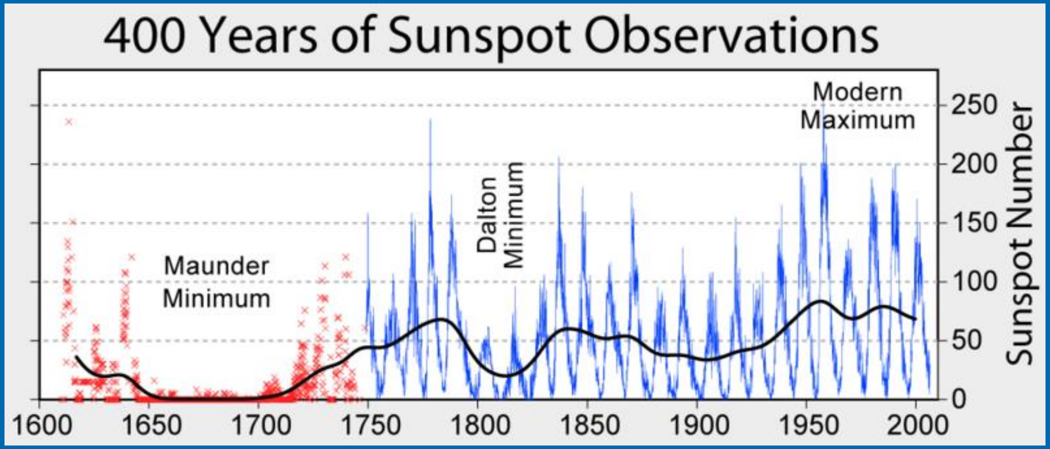
$\mu^2 \simeq .1$ $\kappa = .1$

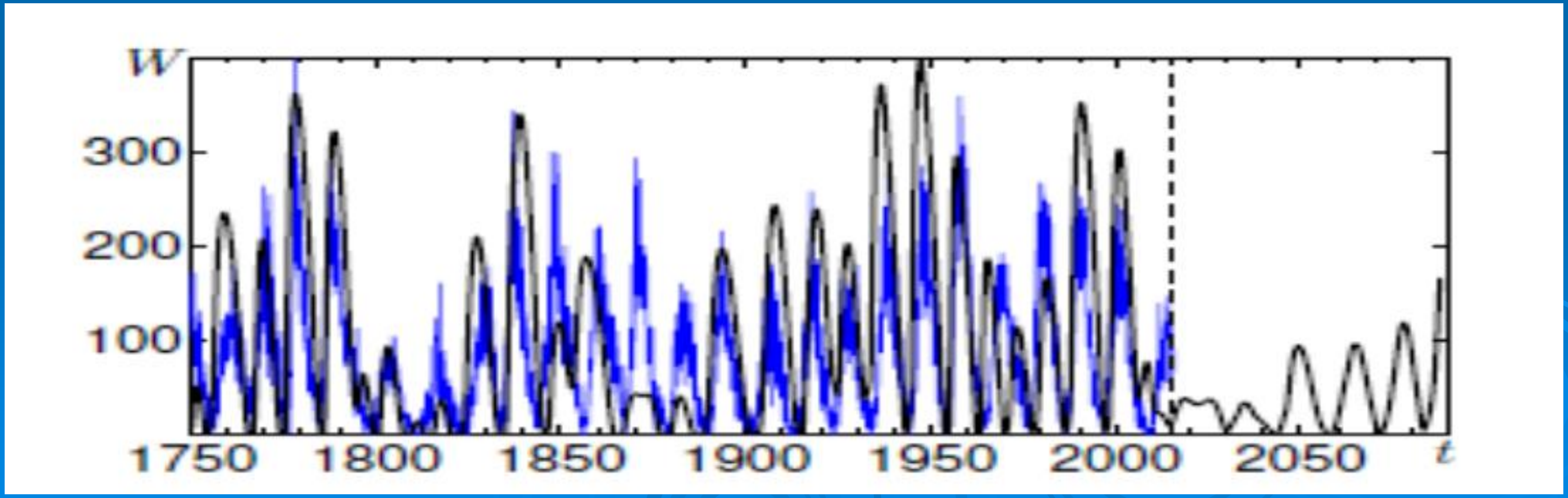
$\sigma = 3$ $T = 6.3$

$R_\alpha = 2$ $\xi = l\sqrt{2}/R_\odot \approx .3$

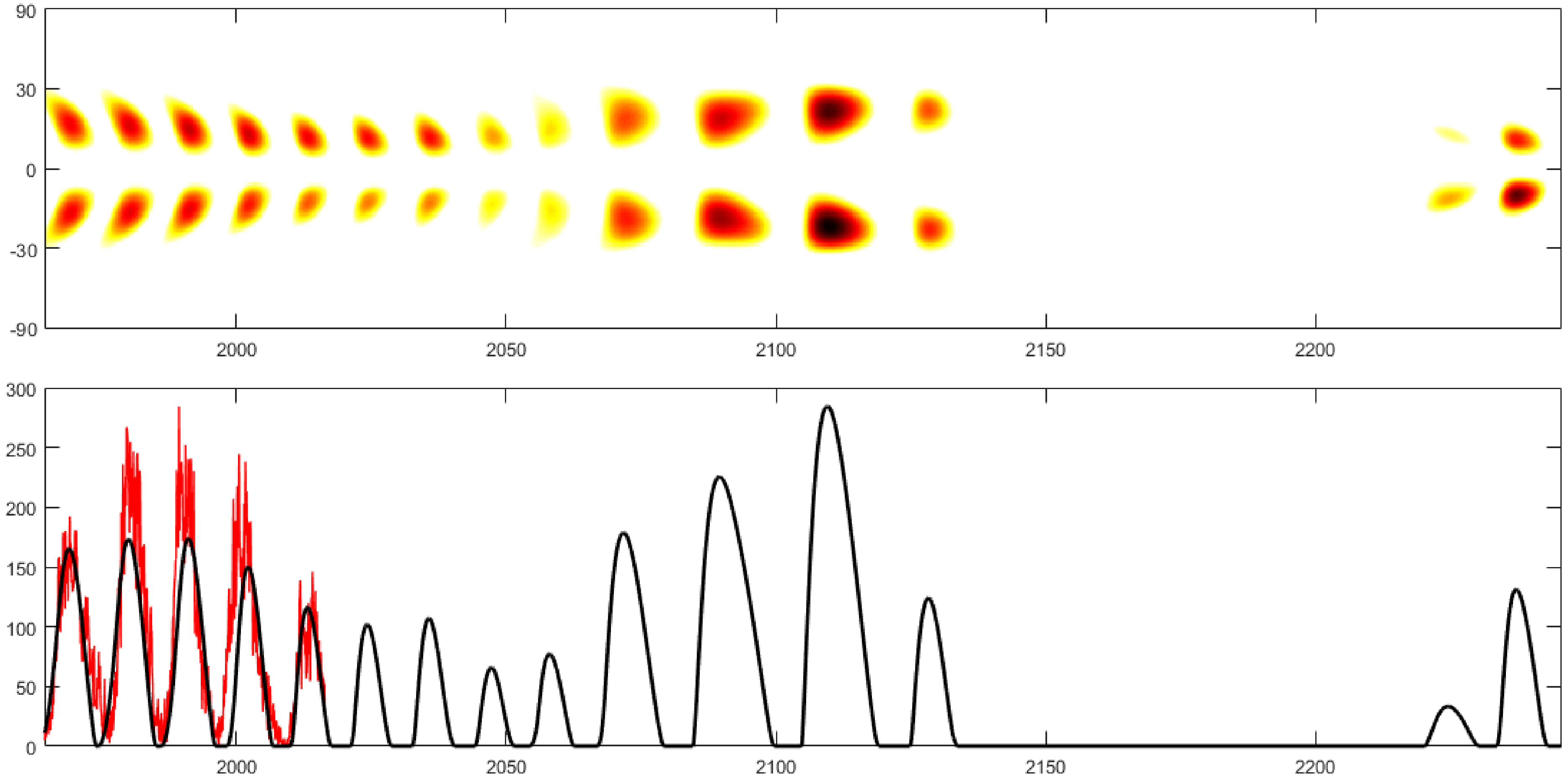


RESULTS (Ya. Kleeorin & ell. 2016)

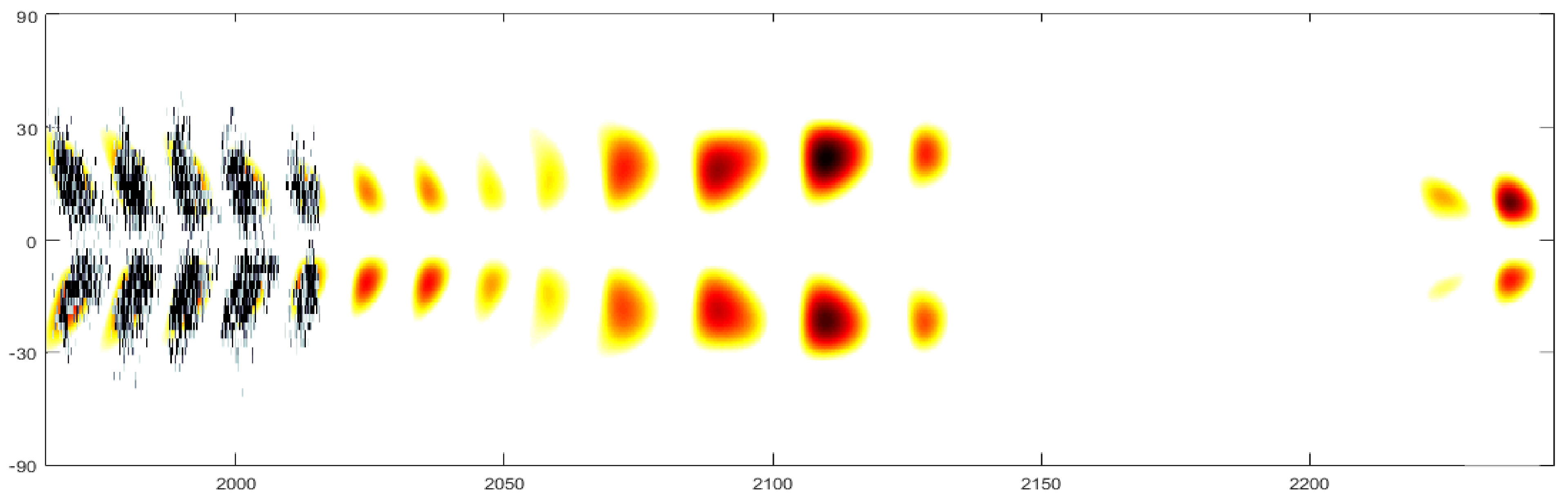




RESULTS: 100-200 years prediction of solar activity

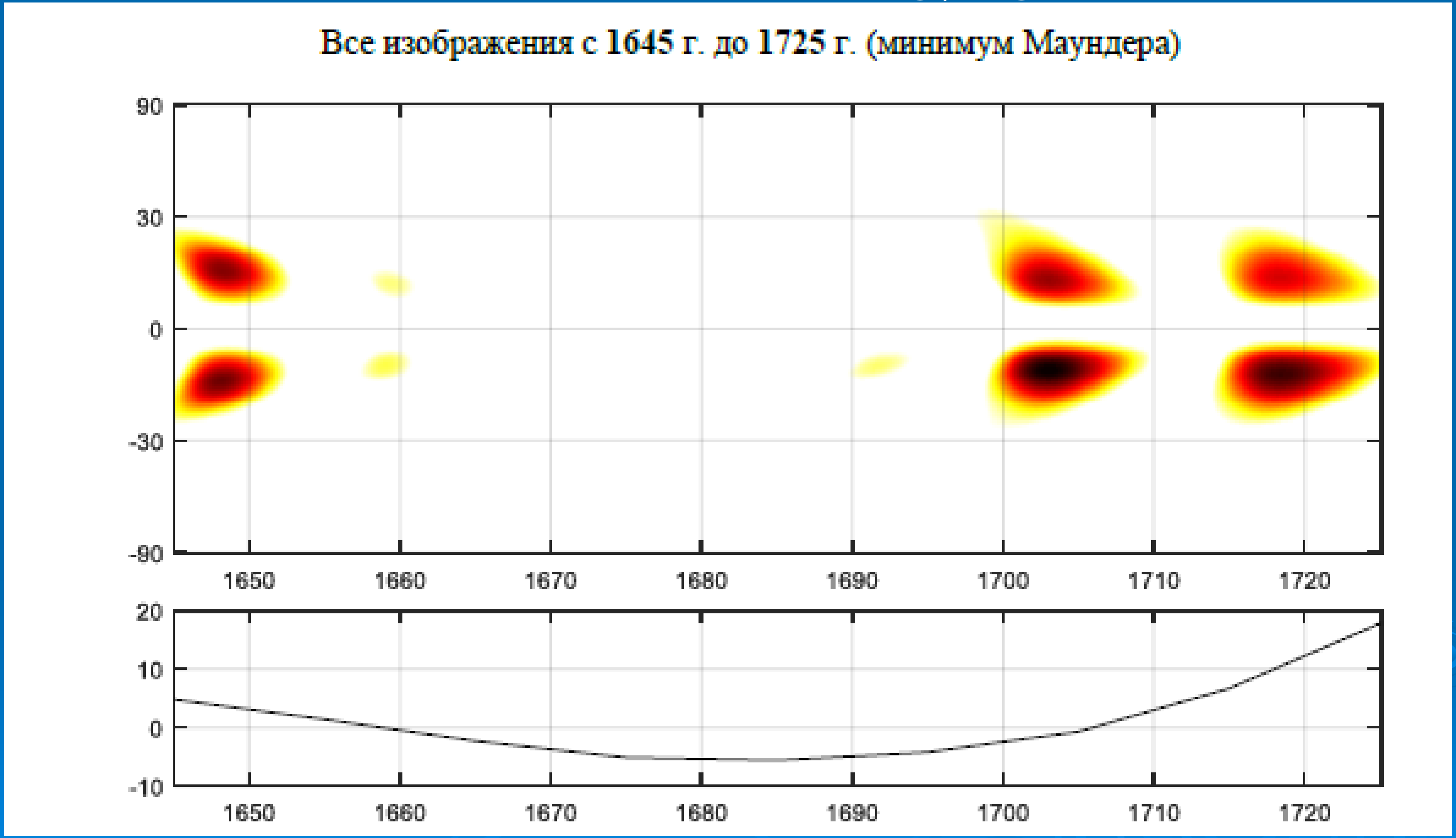


RESULTS: 100-200 years prediction of solar activity $R_{\odot}/L_{\odot} \approx \mu^{-1}$



(Safiullin & all 2018)

RESULTS: 100-200 years prediction of solar activity $R_\odot / L_\odot \approx \mu^{-1}$



(Safiullin & all 2019)

STARS

$$B_s \approx B_{s2}\left(\sqrt{\frac{\bar{\Omega}_*^3}{\bar{\Omega}_{cr}^3}}-1\right)^{\frac{1}{2}}$$

Таблица 2

Спектрал. тип	F0	F5	G0	G2	G5	K0	K5	M0	M5
$M_*/M_\odot$	1.7	1.3	1.1	1	0.91	0.78	0.7	0.47	0.21
$R_*/R_\odot$	1.3	1.2	1.05	1	0.933	0.85	0.74	0.63	0.32
$\eta_T^{(*)}/\eta_T^{(\odot)}$	-----	1.89	1.31	1	0.74	0.48	0.45	0.2	$\approx .5 \div 4$
$\rho_*/\rho_\odot$	0.77	0.75	0.95	1	0.929	1.27	1.73	1.88	6.4
B_0 (Гаусс)	-----	75	67	55	51	41	35	23	217
$\bar{T}$	1.25	1.13	1.024	1	0.971	0.907	0.763	0.678	0.540
B_{s2} (Гаусс)	-----	23	56	74	118	180	345	3,560	188×10^3 – ?
μ_*	∞	6.18	4	3.34	3.16	3.12	3.47	2.26	1
$\bar{H}_*$	0	0.649	0.877	1	0.986	0.909	0.712	0.695	1.069
$\bar{\Omega}_{cr}$	∞	1.238	0.470	0.276	0.210	0.160	0.245	0.114	0.543

STARS

$$B_s \approx B_{s2}\left(\sqrt{\frac{\bar{\Omega}_*^3}{\bar{\Omega}_{cr}^3}}-1\right)^{\frac{1}{2}}$$

Таблица 3

Звезда	Спектра-льный тип	$\bar{\Omega}_*$	B_s (кГс,наблюдения)	B_s (кГс,теория)
Солнце	G2 V	1	1.5	0.193
HD 115383	G0 V	5.2	1.0	0.335
HD 20630	G5 V	2.7	1.8	0.792
HD 131511	K1 V	2.82	1.7	1.5
HD 26965	K1 V	0.686	1.7	0.506
HD 185114	K1 V	0.933	1.36	0.651
Gl 171.2 A	K5 Ve	13.7	2.8	7.0
EQ Vir	K5 Ve	6.51	2.5	4.0

THE END